Dynamic Lag Structure of Deposits and Loans Interest Rates and Business Cycles Formation

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Abstract: The purpose of the paper is to study the dynamic structure behavior of depositors, banks, and investors during business cycle by dichotomizing the money market into two markets of: saving-deposit and investment-credit. The empirical results show that in United States, the banking system creates fluctuations in money sector and real economy as well through interest rates. Short-term interest rates are the source of oscillation and this source of oscillation is emanated from interest rates' effects to real sector, and form business cycles in the economy. The results also show that though the source of fluctuations in real economy comes from short term interest rates, however, medium and long terms interest rates dampens real economy fluctuations.

Keywords: Usury-free banking, Lag structure, Business cycle, Interest rate, Banking Sector.

1 Introduction

Many **c**ountries have experienced business cycles in the economy with a sinusoidal trend. These fluctuations also spread in other countries due to international transactions of goods and flows of capital. As a result, the price signals of commodities and various capital yields create the global flow and the movement of business cycles. Various business cycles happen weekly, monthly or seasonal, which were caused by inventory fluctuations and last less than half a decade. These cycles are called "Kitchin" cycles, (Kitchin, (1923)). It is not a significant point of discussion of this paper. The focus of this paper is on the fluctuations in the economic or business cycles, which take place in a decade and turn the economy from prosperity to recession and crisis, and usually it takes another decade to recover the economy. The large cycles, which were caused by structural changes, are known as "Kuznets" cycles, (Kuznets (1930)). The longer fluctuations caused by science and technological developments that take about half a century to complete, are called "Kondratieff" cycles, (Kondratieff and Stolper (1935)). Sometimes, the overlapping of these cycles with usual business cycles creates large global crises such as the crises of 1929 and 2008.

Behaviorally, this instability is due to man's nature to spend exuberantly. The prosperous ones spend extravagantly but the limited available sources do not let him to continue. Therefore, it has to stop during the economic prosperity. That is because resources do not increase proportional to people's economic activities. The scarcities of resources cause price increase and decrease of profit margin, which is the beginning of recession and the turning point of business cycle. This reasoning is actually emanates from human being nature; but technically speaking, the inconsistent timed-structure of loans and deposits forces banks (as intermediary between monetary resources and real sector activities) to experience different losses and profits at different times. In this paper, we are going to show that the time structure of loan and deposit burden financial loads to banks and form monetary oscillation. At the next stage, monetary oscillation affects real economy. Our methodology in this paper is to dichotomize banking activities into two markets of 1: saving supply and bank's deposit demand market and 2: investment demand and bank's credit supply. First market forms deposit interest rate and second market forms credit interest rate. In analysis of these two rates, we will show that if these two types of interest rates were time-inconsistent, then monetary fluctuation starts. As a result, the fluctuation is transmitted to real sector through saving and investment functions. Empirically we will test our hypothesis through estimating the characteristic roots of difference equations we derived from our theoretical analysis using USA 10 different interest rates.

2 Business Cycle Theories

This phenomenon was realized for the first time by Juglar (1862) for spans of 8-11 years. Later, several theories were introduced about business cycles, which studied business cycle from different points of view. Schumpeter (1954) described the four stages of business cycle. The first stage of prosperity wherein there is an increase in production and prices and decrease of interest rate. During the second stage of recession wherein the production and prices decrease but interest rate increases until the third stage is reached of crisis due to collapse in stock market and bankruptcy. The recovery begins during the fourth stage, which is accompanied by stock exchange prosperity and the increase of output, demand and prices.

Goodwin (1949, 1991) believes that the reason for business cycles is the gap between income distribution between the profit of investors of economic firms and the earnings of labor force. When the economy has a high employment rate, the labor demand increases but the workers cannot ask for higher wages as the labor contracts are annual or have fixed periods and wages can only be changed after the end of the contract period. The reverse happens during recession. Therefore, the income of the labor force is adjusted with the income of capital factor after a time lag, which creates a cyclical behavior for matching production with consumption and ultimately shapes the cycle. Although Goodwin's theory has time dynamism, but mathematically its simple form gives a first order difference equations. Although these equations have time ascending or descending trend, and are converging or diverging trends, but cannot obtain time oscillatory trends. The difference equation should be at least of second order to create sinusoid trends. Some economists believe that the reason for business cycles is technological shocks (Real Business Cycle Theory. Kydland and Prescott (1982)), and the others believe that they are created by political parties and political decision cycles, (Political Business Cycle. Partisan Business Cycle, Michal Kalecki). Marxists believe it as the essence of capitalism, and neoclassic economists believe that the decrease of labor purchasing power is the reason for capitalistic crises.

By scrutinizing the nature of unstable and sinusoid movement of economic activities, we realize that the nature automatically leads to situations in which economy turns from prosperity to recession and crisis and back to prosperity. Some believe the reason behind this sinusoid movement is inventory operation and supply flow. That is, when the economy produces more than consumption, goods are accumulated in the warehouses and the producer has to decrease price to sell the inventory. The decrease in price and the large accumulated inventor loosens production in turn compelling the producer to decrease production and thus decreases the income of factors of production (labor, capital and others). That is to say, income at macro level will decrease the demand for goods and services. The lower demand will decrease price and economic firms will face more recession. This phenomenon goes ahead until recession changes to crisis. During this stage, production continues to decrease, but practically the trend of price decrease continues or stops. Price reaches production cost and therefore, it is not possible for the producer to decrease the price below the production cost leading to cessation of production and many inefficient firms may become bankrupt and total supply in the country will decline. The decrease in supply in turn is followed by increase in price and persuades production, which increases income of factors of production in the next stage and is followed by more price increase. As a result, the economy may move from crisis to prosperity until we reach the beginning of next business cycle. This is followed by recession and the cycle repeats itself.

According to Keynesian Economists such as Samuelson (1939), fluctuation in total demand causes the economy to reach equilibrium in a short period, which is not in equilibrium at full employment. The motivation to obtain full employment in equilibrium and the inefficient excessive use of resources and factors of production and production capacity will lead to business cycles. Keynesian theories believe that the lack of enough effective demand in the economy is an indigenous cause for crises while Classic and Neoclassic economists believe exogenous factors are the causes of business cycles. They believe supply will create its own demand (Say's Law). According to these two views, interference of government policies will have a positive or a negative effect in avoiding crises resulting in different policy. The former offers financial policy and the latter offers monetary policy. Paul Samuelson's (1939) Oscillator Model describes Keynesian analysis based on multiplier effect (on consumption) and accelerator (in investment) which create cycles through changes of total demand components. The struggle between Keynesian and Classic economists can be introduced in this discussion. Keynes (1936) believed when the economy is in liquidity trap, or there is no coordination between saving and investment, in order to obtain equilibrium in money market, it is necessary to obliterate interest rates. In this situation, it is not possible to use monetary policies to overcome crisis because interest rate is zero. Essentially, the liquidity trap occurs due to the poor relationship between return rate of production in real sector and interest rate. That is to say, the interest rate has decreased to a very low figure, regardless of productivity rate. One of the reasons behind this situation is the increase of investment risk, which decreases the net investment yield. The point is that, if it were possible to make negative interest rate in the economy, which is practically impossible, monetary policy would be capable of overcoming crisis. Therefore, national policies are recommended which are conducted in Group 20 by injecting about US$5 trillion to overcome the present crisis. This method practically causes the crisis stricken economies to overcome the crisis. In order to achieve this, it is necessary to wait for about half a decade until economic mechanisms lead the economy to prosperity through natural ways. The overview of economic variables in the last decade shows the same in comparison with 1920 crisis. The sever oil and grains prices increase and successive draughts and unexpected ecological and meteorological events had increasing effects on this crisis. Similar conditions of the present crisis were also seen in the crisis of 1920 decade. Also during that period, interest rate was severely reduced, but the economic conditions were so bad that even the low price of capital could not allocate adequate resources for production. Therefore, to recover the global economy we have to wait like the financial policy, which took about a decade to recover the global economy from crisis.

3 Money Market and the Role of Banks in Creation of Economic Cycles

The essential solution to overcome the crisis is to reform of monetary and banking structure. Some theories such as Austrian Business Cycle Theory (Block and Barnett (2007)) suggest that banking structure is one of the factors to create crisis. In spite of excessive Inventory Theory, the variation of inventory is the effect of cycles, not its cause. Unlike the prevailed belief, the recent recession was not because of the recession in America housing market but rather it was the consequence of this crisis. We hypothesize that crisis is a result of structural behavior in money and banking sector. The theory of credit cycles by Irving Fisher (1933) is one of the interesting theories about the cause of business cycles. He believed that credit cycles are the starting reason for economic cycles. Accordingly, the net increase of credits and the debts as a percentage of GDP creates economic prosperity, and vice versa, the decrease of net credit, moves the economy towards recession and crisis. In direction of Fisher theory, Minsky (1992) puts financial instability hypothesis forward and developed Fisher theory by describing credit bubbles and the burst of these bubbles and their effects on economic cycles. He believes that the reason is the accumulated debts to banks. In this connection the Austrian school of thought (Ludwig von Mises and Friedrich Hayek works) can be put forward which believes that the cause of credit changes is the monetary expansionary policies of central banks. This school refers to the role of interest rate as the price of capital for investment and agrees that in an open economy without central bank, interest rate describes the real time preference of borrowers and lenders. Nevertheless, central bank disturbs this equilibrium between them and inevitably creates fluctuations in the economy. When Fisher put forward this theory, the dynamic mathematical tools such as difference equations had not been invented and used yet. If difference equations which was set forth in 1950s (by Baumol (1958), Baumol and Turvey (1951)) were introduced into economic analysis a few decades earlier, by using this tool, Fisher could conclude the occurrence of crisis, as he explained the richest monetary theory of The Quantitative Theory of Money with the help of Balance Law from physics.

Recent researches such as Beaudry and Guay (1996) have also touched some aspects of the relation between interest rate and business cycle; but no one of them looks at the problem as is discussed in this paper. Beaudry, and Guay (1996) document the extent to which the predictions of standard Real Business Cycle (Kydland, and Prescott (1982)) models are incompatible with observed movements in real interest rates. That is why this study just tries to show the co-movements between interest rates and output and not the causes. Theoretically, world real interest rate is an important mechanism by which foreign shocks are transmitted to small open economies. Blankenau et al (2001) express that while the world real interest rate is potentially an important mechanism for transmitting international shocks to small open economies, much of the recent studies show that this mechanism has little effect on output, investment, and net exports. Although their study confirms the effect of interest rate shock on real economy, but looks for transmission channel. Furthermore, Ivanova et al (2000) have studied the comparative performance of a number of interest rate spreads as predictors of the German inflation and business cycle in the post-Bretton Woods era.

Generally, contemporary macroeconomics discusses in a single money market, we will further divide it into two separate markets where banks operate as intermediary between them. Dichotomization of bank’s activities into two markets of “saving supply versus bank’s deposit demand” and “investment demand versus bank’s credit supply” is new to literature. The only seen background can be found in Bidabad (2006) as an equations block in macroeconometric model of Iran which used this dichotomization essentially for the first time. In other words, the demand of the bank for deposits is at one side of the bank, which intersects with the supply of deposits and fixes interest rate, which is called deposit interest rate. On the other hand, bank creates another market by supplying funds for credit finance and its intersection with demand for credit facilities creates the interest rate for credit facilities. That is to say, bank stands between two markets of supply and demand of money funds. Now suppose the consumption increases, the deposit supply will fall. This will increase the deposit interest rate. The increase in deposit interest rate cannot instantly increase credit interest rate because credit contracts have been fixed for a period and bank has to wait for the duration of the contract before increasing the rate in new contracts for credit facilities and subsequently, the interest rate for credit facilities will increase in the economy. A bank may face losses during this period and after a time lag will compensate it by the increase of credit facility rate. Although this lag is not quite visible for people, from economic point of view it creates a special dynamic relationship between supply and demand for capital. It can mathematically be shown that because of this lag, the relationship between these two variables is a second order behavioral difference equation. Second order difference equations have a wavy character, which creates cycles. Therefore, practically, the cycles created in the saving and families' consumption will transfer into the investment and production sectors.

In other words, all fluctuations seen in real sector of the economy are induced by fluctuations in the money market. Monetarist theories have been carried out on this subject that if in the economy, fluctuations of the money sector decrease, many fluctuations of the real sector will tend to stabilize. The most important effect of elimination of usury or interest is the direct bridging of the investment sector to saving sector of the economy, (Bidabad (2004)). When banks try to maximize profit through optimizing their behavior, the intermediate sector (bank) acts as a separate sector of the economy and the differences created by them between interest rates of supply and demand of funds, create fluctuations in financial markets. Since loan contracts have maturity periods, changes in supply (sources) or demand (uses) sides of funds are transferred to the other side. This time lag creates continuous fluctuations in financial markets. This analysis can be seen in figure 1. Where:

S^B	Loan supply by banks
S^S	Fund supply by depositors
D^L	Loan fund demand
D^B	Bank demand for funds
r^S	Saving interest rate
r^L	Loan interest rate
m^S	Amount of saving
m^B	Amount of loans
R	Bank’s revenue

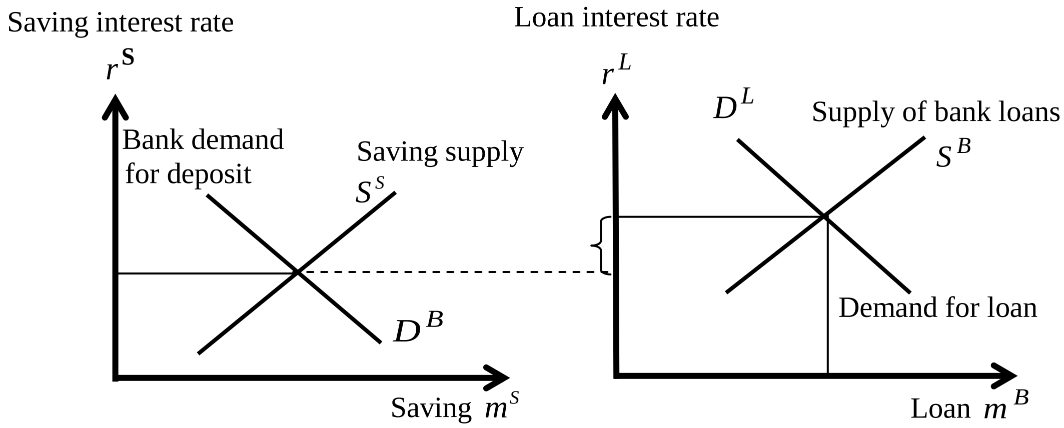


Figure 1

Bank's revenue at time t is equal to:

$$R_t = m_t^B r_t^L - m_t^S r_t^S \quad (1)$$

At equilibrium, we have:

$$m_t^B = D_t^L = S_t^B \quad (2)$$

$$m_t^S = D_t^B = S_t^S \quad (3)$$

Now suppose a new condition in which demand for loans decreases and D_t^L moves to left side to D_{t+1}^L. In the new equilibrium, if bank's revenue turns negative, we have:

$$r_{t+1}^L < r_t^L \quad (4)$$

Or:

$$R_{t+1} = m_{t+1}^B r_{t+1}^L - m_t^S r_t^S < \circ \quad (5)$$

Therefore, in respect of time-loan contracts of bank, bank has practically to compensate losses during t+1 period from other sources until the next period when D^B curve moves to the left hand side. That is to say:

$$r_{t+2}^S > r_{t+1}^L \quad (6)$$

$$R_{t+2} = m_{t+1}^B \ r_{t+1}^L - m_{t+2}^S \ r_{t+2}^S > \circ \quad (7)$$

By generalizing this hypothesis we clearly see that whenever shocks occur in deposit supply or demand for banks' credit facilities (loan), because of time contracts, these shocks will be transferred to the other market in the next period and the fluctuations transfer from market to market alternatively and permanently fluctuates other related markets.

By considering the sign of three equations of (1), (5) and (7), we can clearly see that the behavior of variable R is alternative in different time periods. The behavior of the two markets described above can be drowned according to Cob-Web model which creates different fluctuation according to the gradient of different parts of supply and demand curves.

The interest rates in the two markets are as follows:

$$r_t^S = r^S(m_t^S) \quad (8)$$

$$r_t^L = r^L(m_t^B) \quad (9)$$

According to the above assumptions if we adjust the relationship of the two markets with one time-lag, we have:

$$m_{t+1}^S = f(m_t^B) \quad (10)$$

By replacing (8) and (9) in (10), we have:

$$r_t^S = r^S(f(m_{t-1}^B)) = r^S(f(r^{L^{-1}}(r_{t-1}^L))) \quad (11)$$

In other words, the interest rate in the deposit market is a function of interest rate in loan market in the last period. The adjustment takes place when the return movement occurs in the next period which means that the interest rate of loan market is itself a function of interest rate of deposit market in the previous period, or:

$$m_{t+1}^B = g(m_t^S) \quad (12)$$

By replacing (10) in (12), we will have:

$$m_{t+1}^B = g(f(m_{t-1}^B)) \quad (13)$$

This is a second order difference equation, which is characterized to fluctuate easily in time. This is also true about interest rates. By replacing (12) in (10), we have:

$$m_{t+1}^S = f(g(m_{t-1}^S)) \quad (14)$$

This equation similar to equation (13) can be completely oscillatory. By replacing (12) in (9), we have:

$$r_t^L = r^L(g(m_{t-1}^S)) = r^L(g(r^{S^{-1}}(r_{t-1}^L))) \quad (15)$$

Since equations (15) and (11) are function of m_{t-1}^{S} and m_{t-1}^{B} these two variables can completely be oscillatory according to (14) and (13). Therefore, interest rate similar to the amount of deposits (savings) and loans in both loans and deposits markets can fluctuate.

4 Transfer of Fluctuations from Money sector to Real sector

Although the transfer of monetary fluctuations to real sector can be clearly seen and understood, but in order to clarify the subject, we consider the equilibrium at macro level and its relation with interest rate fluctuations induced by the banking behavior. According to national accounting relationship we can write:

$$\begin{aligned} gdp &= con + inv + gov + ex - im \\ gde &= con + sav + tax + tr \\ gdp &= gde \end{aligned} \qquad (16)$$

In which:

gdp	Gross domestic product
gde	Gross domestic expenditure
con	Consumption
inv	Investment
gov	Government expenditure
ex	Export
im	Import
sav	Saving
tax	Tax
tr	Transfer payment to outside

By solving equation (16), the equilibrium condition in macro economy will be obtained:

$$(inv–sav) + (gov–tax) + (ex-im–tr) \qquad (17)$$

Foreign exchange markets relate international capital markets to real sector through foreign exchange rates and financial derivatives such as options, futures, forward contracts and time deposit certificates where these markets also oscillatory affect the real sector through interest rate similarly. However, for simplicity, we do not consider it and according to equations (15) and (11), we will only consider the two variables of investment and saving as functions of interest rates of saving deposits and loans (r^S and r^L). In other words, the equilibrium condition in the economy in time t will be as follows:

$$(\mathrm{inv}_t(r_t^L) - \mathrm{sav}_t(r_t^S)) + (\mathrm{gov}_t - \mathrm{tax}_t) + (\mathrm{ex}_t - \mathrm{im}_t - \mathrm{tr}_t) = 0 \qquad (18)$$

By replacing r_t^S and r_t^L from equations (15) and (11) in equilibrium condition, we will have:

$$\left(\mathrm{inv}_t\left(r^L(g(r^{S^{-1}}(r_{t-1}^L)))\right) - \mathrm{sav}_t\left(r^S(f(r^{L^{-1}}(r_{t-1}^L)))\right)\right) + (\mathrm{gov}_t - \mathrm{tax}_t) + (\mathrm{ex}_t - \mathrm{im}_t - \mathrm{tr}_t) = 0 \qquad (19)$$

If the government has balanced fiscal policy which means (gov_t - tax_t)=0 and the trade balance is also balanced (ex_t - im_t - tr_t)=0, again the equilibrium (19) is a second order difference equation which can lead the economy into disequilibrium in different times. In other words, the mathematical behavior of equation (19) will be different regarding the behavioral characteristic of saving and loan contracts and the reaction of depositors and borrowers (investors) to interest rates of deposits and loans, which can show any kind of oscillatory behavior. Nevertheless, this behavior cannot be expanding (diverging) or dampening (converging) forever, because disequilibrium tends to zero in long run. Therefore, necessarily even if the equilibrium is a moving equilibrium, it should oscillate around its long run equilibrium and this phenomenon creates economic cycles.

It may be necessary to brief the same by using fiscal policy (gov_t - tax_t). According to Keynes theory, we can compensate disequilibrium in equation (18). But by injecting financial resources to the economy equal to (gov_t - tax_t)>0, the excess government budget at time t *Ceteris Paribus* will be government debt to the central bank at time t+1 and disrupts budget balance. As a result, the government has to levy more tax to compensate this deficit in the next period, which leads to recession and disequilibrium and fluctuation in the next period. The same is true about the last parentheses of equation (19) about trade balance (ex – im – tr). By creating export incentives and import barriers for imports at time t, government increases the trade balance equal to (ex – im – tr)>0, and by increase of exchange rate for the period t+1, exchange stability will be disrupted and we will face deficit in foreign trade which will have negative effects on the economy and creates fluctuations.

5 Capability of Second Order Difference Equations in Explaining Fluctuations

In order to clarify the second order difference equations' behavior further, we will brief the time trend dynamism of linear non-homogenous difference equation with fixed coefficients. Consider a linear non-homogeneous second order difference equation with constant L and fixed coefficients of "a" and "b":

$$y_{t+2} + ay_{t+1} + by_t = L \qquad (20)$$

According to the amounts of "a" and "b", the time trend of this equilibrium can be linear or non-linear for each Particular Solution. By foregoing L for the time being, the Complementary Function will be as follow:

$y_{t+2} + ay_{t+1} + by_t = 0$ (21)

This difference equation has the following second order Characteristic Equation with roots x', x":

$x^2 + ax + b = 0 \quad x', x'' = (-a \pm \sqrt{a^2 - 4b})/2$ (22)

The Final Solution is in the form of the sum of “Particular Solution” (y_p=M) (by solving the non-homogeneous equation) and “Complementary Function”: $y_t = A(x')^t + B(x'')^t + y_p$

In case, the equation has two real positive or zero roots, when both of these roots are real and less than one, by increase of t, y_t tends to constant M (figure 2). But if x'=1 and x"<1, when t increases, y_t tends to A+M (figure 3). If x"=1 and x'<1, when t increases, y_t will finally tend to B+M. If one of the roots is bigger than the other (x'>1) and B and A are both positive, by increase of t, y_t will increase (figure 4). If A<0 and B >0, the result depends on whether x' or x" is larger. At the preliminary stages, by increase of t, the amount of y_t is more affected by the term whose root is larger (figure 5).

Figure 2

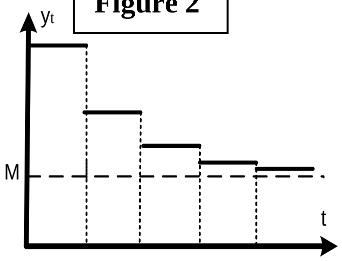


Figure 3

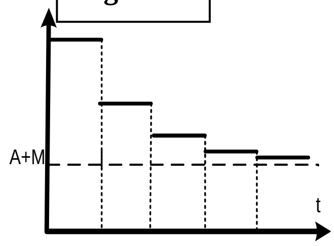


Figure 4

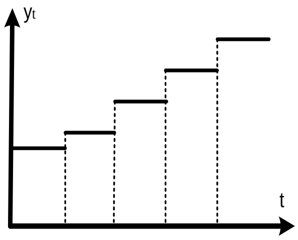
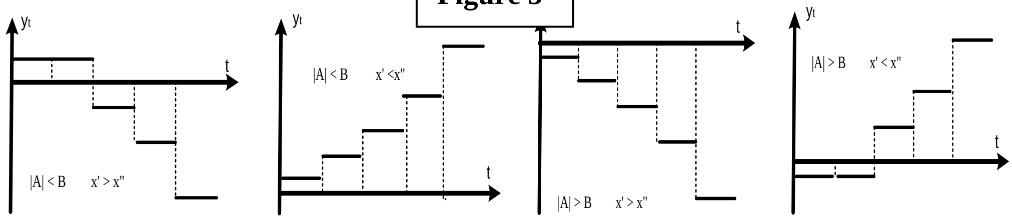


Figure 5



In case, the two roots are real and negative (or zero), when x", x'<0, when t is even, $(x'')^t$ and $(x')^t$ will be positive, and when t is odd, these items will be negative. This causes y_t to oscillate in successive periods. If the absolute value of both roots are less than one, 0<|x'|, |x"|<1; which means: -1<x', x"<0, by increase of t, $|A(x')^t|$, $|B(x'')^t|$ will reduce and the amount of y_t tends towards M and finally will fit on line y_p= M (figure 6). If x'= -1 and -1<x"<0, $A(X')^t$ does not depend on the amount of t, but only on its being odd or even. When t is even, the last expression is equal to A and when it is odd, it will be equal to –A. Also by increase of t, the amount of $B(x'')^t$ will decrease and y_t oscillatory tends to $A(x')^t$+M and $A(-1)^t$+M (figure 7). When x',x"<-1 , y_t fluctuation will increase by the increase of t and the root with higher absolute value, will have a more effective role in fluctuations of y_t (figure 8).

Figure 6

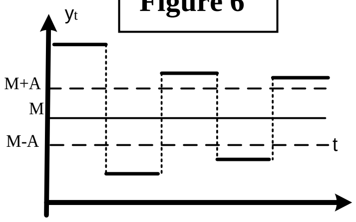


Figure 7

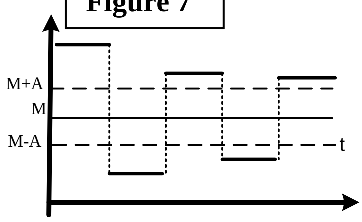
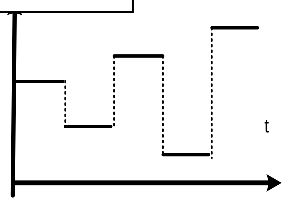


Figure 8



In case, one root is positive and the other is negative, the absolute value of the larger root will have more effects on y_t. The following cases may occur:

If 1>x'>0 and |x'|>| x", y_t will finally tend to M (figure 2).

If x'=1 and |x'|>|x"|, y_t tends to A+M (figure 3).

If x'>1 and |x'|>|x"|, the increase of t will increase y_t (figure 4).

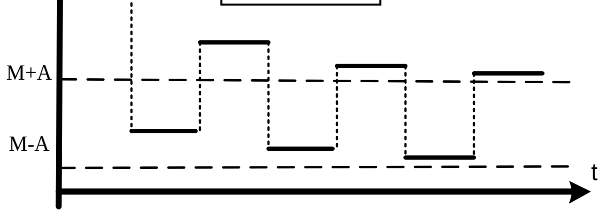
If -1<x'<0 and |x'|>|x"|, by increase of t, the amount of y_t will fluctuate and finally tend to M (figure 6).

If x'=-1 and |x'|>|x"|, by the increase of t, y_t will oscillate around A+M and –A+M (figure 9).

If x'<-1 and |x'|>|x", y_t will have expanding fluctuations in relation with M when t increases (figure 8).

If x'=-x", changes in y_t depends on the sign of A, B and larger absolute value of A in relation with B and vice versa.

Figure 9



In case, the roots are double and real, the solution of the difference equation will be as follows:

$y_t = Ax^t + Btx^t + M$ (23)

If x>1, Btx^t will increase by the increase of t and will have an expanding time trend.

If x=1, and $Btx^t = Bt$, we will have a similar case to previous one.

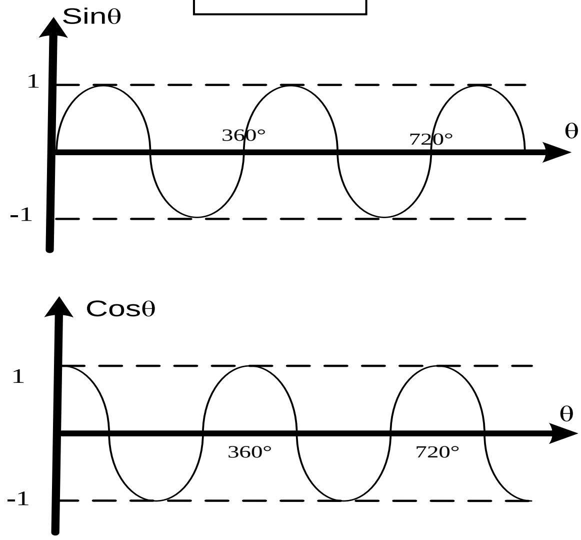
If 0<x<1, Btx^t tends to zero when t increases. This trend is not compensated by the expanding trend of t and when t increases, x^t will be more than t and therefore, the term Btx^t will tend to zero.

If -1<x<0, we have a trend similar with the case 0<x<1, except that the term Btx^t has a fluctuation trend which tends to zero.

If x=-1, the trend of Btx^t is similar with x=1, but with fluctuations.

If x<-1, the result is similar with x>1, but with fluctuations.

Figure 10



In case, the roots are imaginary or complex, the solution of the difference equation will be:

$y_t = D^t[E \, Cos \, (tR)+F \, Sin \, (tR)] + M$ (24)

Where the roots of the complex characteristic equation will be c±di and its module will be D=$\sqrt{c^2+d^2}$. The term in the bracket has an iterated fluctuation in each 360°. In other words, y_t has a cycle equal to t=360°/R. Since Sin R=d/D, if for example R=60°, the range of the cyclical movement will be t=360°/60°.

For, θ=tR the curves Sinθ and Cosθ in the figure 10 have symmetrical cycles. Since amount of t is discrete, t may not be located at the beginning or middle of those cycles; therefore, figures 11, 12 and 13 look unbalanced. The term in the brackets of equation (24) has smooth fluctuations, but when reaches to power t, if D>1, by increase of t, D^t will increase and multiplied by the amount of the brackets in equation (24) will increase the fluctuations (figure 11).

Figure 11

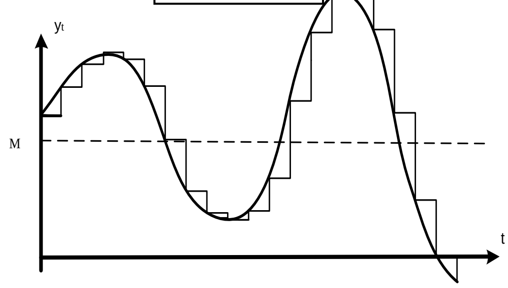
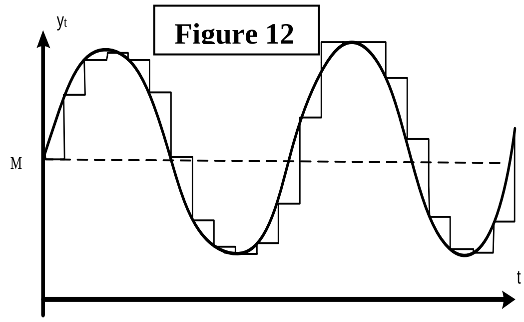


Figure 12



If D=1, D^t will also be equal to 1 and will have no effects on the amount of the bracket and y_t will have a monotonous (smooth) fluctuations (figure 12).

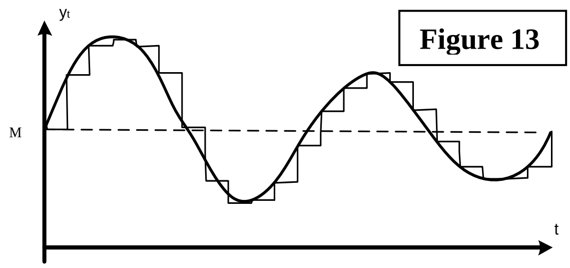


Figure 13

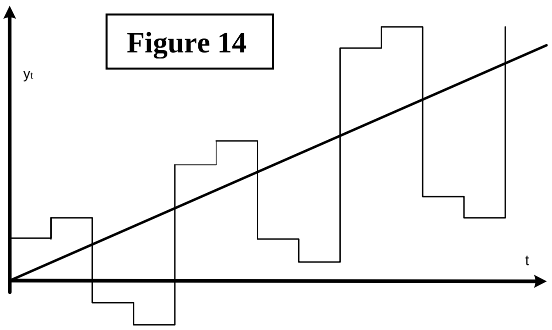


Figure 14

If D<1, when t increases, D^t decreases and fluctuations of y_t will tend to zero (figure 13). If we have a Particular Solution, (this solution may be in the form of M, Mt or Mt^2), we define y_p as the temporary equilibrium of y_t. But if the Particular Solution is not fixed, the equilibrium will be moving. For example, the Particular Solution can be in the form of $y_p=bt$ where the difference equation has no temporary equilibrium and bt is a moving equilibrium (figure 14).

In general, for a second order homogenous difference equation with constant coefficients such as:

$$y_t + by_{t-1} + cy_{t-2} = 0 \qquad (25)$$

With the help of figure 15 we can describe the time trend of y_t by considering the coefficients of difference equation (b, c) and the roots of characteristic equation.

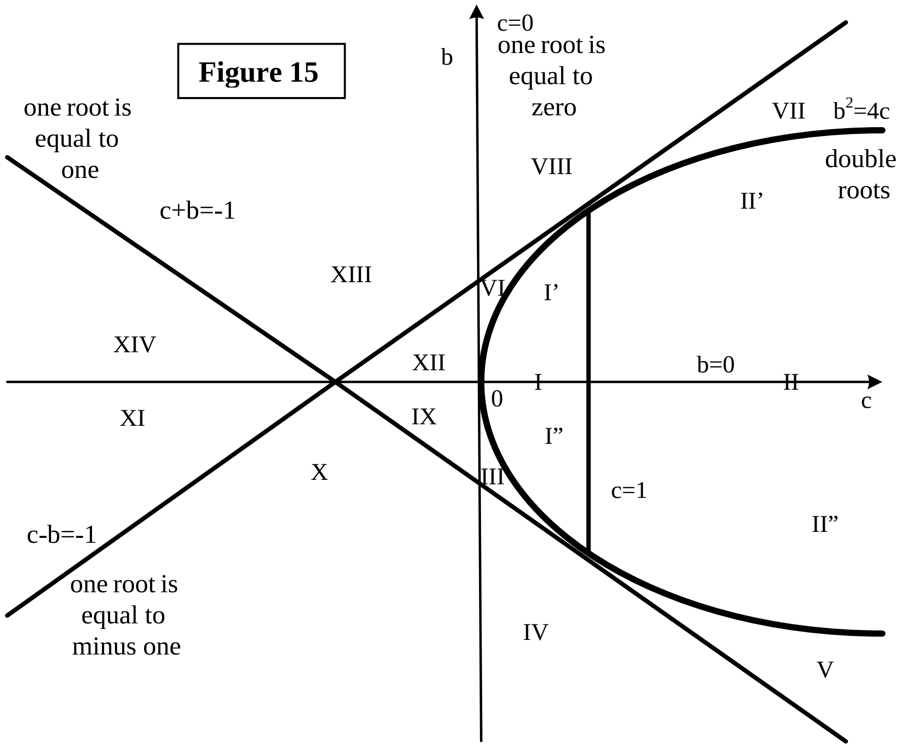


Figure 15

In figure (15):

Area I: includes (I", I'): oscillatory dampening fluctuation (complex roots, module<1)

Area II: includes (II", II'): explosive oscillatory fluctuation (complex roots, module>1)

Area III: dampening and non-oscillatory (real roots, less than one)

Area IV: finally dampening and without oscillation (real roots, both positive, one larger than and the other less than one)

Area V: non-dampening and without oscillation (real roots and larger than one)

Area VI: oscillatory dampening (real roots, both negative with absolute values less than one)

Area VII: explosive with oscillation (real roots, both negative with absolute values larger than one)

Area VIII: finally explosive with oscillation (real roots, both positive and less than one)

Area IX: dampening without oscillation (real roots, both positive and less than one)

Area X: finally explosive without oscillation (real roots with opposite signs, the positive root is larger than one and the absolute value of the negative root is less than one)

Area XI: finally explosive without oscillation (real roots, with opposite signs, the absolute value of both roots are larger than one and the absolute value of the positive root is larger)

Area XII: dampening with oscillation (real roots with opposite signs, the absolute value of both roots less than one)

Area XIII: finally explosive with oscillation (real

roots with opposite signs, the absolute value of the negative root is larger than one and the positive root is less than one)
Area XIV: finally explosive with oscillation (real roots with opposite signs, the absolute value of both roots are larger than one and the absolute value of the negative root is larger than the positive root)

In analyzing the time trend of y_t in a second order difference equation, the characteristics of being real roots, complex roots and double roots, their signs, the absolute value of them and being larger or smaller than one, are critical. In figure 15, many other areas can be enumerated and studied like all the points located on drawn lines and curve, or at the intersection of them and so on, but we stop here [4].

It may be noted that all the above analyses are only about second order non-homogeneous difference equations with a constant term and fixed coefficients. Nevertheless, if the constant term is a function of time or the difference equation is not linear, or has a higher order, or the coefficients are functions of time, y_t will have different trends which create moving equilibriums with various fluctuations. The mathematical behavior of this kind of equations can be coinciding with difference equation in the previous discussion about interest rates and deposit and loan sources which create fluctuations in money and capital markets that can easily fluctuates the supply and demand in real sector through investment and consumption demands functions.

6 Empirical Investigations

To test the time structure of the interest rate the following interest rates were selected for a long period since 1948 till present (International Financial Statistics, Country Note 2010, USA):

Discount Rate (End of Period): Rate at which the Federal Reserve Bank of New York discounts eligible paper and makes advances to member banks. Borrowing from a Federal Reserve Bank may take the form either of discounts of short-term commercial, industrial and other financial paper or of advances against government securities and other eligible collateral. Federal Reserve advances to or discounts for member banks are usually of short maturity up to fifteen days.

Federal Funds Rate: Weighted average rate at which banks borrow funds through New York brokers.

Commercial Paper Rate: Rate on three-month commercial paper of nonfinancial firms. Rates are quoted on a discount basis and interpolated from data on certain commercial paper trades settled by the Depository Trust Company.

Treasury Bill Rate: Weighted average yield on multiple-price auctions of 13-week treasury bills.

Certificate of Deposit Rate: Average of dealer offering rates on nationally traded certificates of deposits.

Lending Rate: Base rate charged by banks on short-term business loans.

Mortgage Rate: Contract rate on 30-year fixed-rate first mortgages.

Government Bond Yield: Long-Term and Short-Term: Yield on actively traded treasury issues adjusted to constant maturities. Yield on treasury securities at constant maturity are interpolated by the U.S. Treasury from the daily yield curve. This curve, which relates the yield on security to its time of maturity, is based on the closing market bid yields on actively traded treasury securities in the over-the-counter market. These market yields are calculated from composites of quotations obtained by the Federal Reserve Bank of New York. Medium-Term rate refers to three-year constant maturities. Long-term rate refers to ten year constant maturities.

Figure 16 shows the time path of different interest rates during the sample period.

To show the oscillatory natures of the interest rates we need to test that the equations (11) and (15) hold true. That is why these two equations are difference equations with (at least) orders 2 or more. Using data of the Unites States of America for the period of 1948-2009, we try to show that equation (11) and (15) are oscillatory and therefore the source of oscillation will be transferred to equations (18) and (19) which are the equilibrium condition of the macro-economy. We can conclude that the source of oscillation is emanated from interest rates to real sector, and the interest rates are the source of cycles in the economy.

We test the hypothesis that do various interest rates obey oscillatory behaviors? Therefore, ten types of short term, medium term and long-term interest rates were selected. We fit a second order linear non-homogenous difference equation to all 10 interest rates as given by (20). Table 1 shows all estimated parameters are statistically significant and proves that a second order linear difference equation time structure exists for all 10 selected interest rates. Table 2 shows all short-term interest rates' second order homogeneous linear difference equations have complex characteristic roots; but the roots for medium and long-terms interest rates are real. These results prove that the source of fluctuations in real economy comes from short-term interest rates, though medium and long-term interest rates have real characteristic roots and dampening behaviors.

Since values of D in short-term rates are less than or nearly equal to or greater than one, the forms of oscillations will be as shown by figures 11 to 13. On the other side, the medium and long-term interest rates have real characteristic roots, which each one of the pairs is close to one and another is much less than one. Therefore, the medium and long-term interest rates have time path similar to figures 2 and 3 and have dampening behaviors.

Figure 16. Different interest rates time trend. USA (%) for 1948-2009

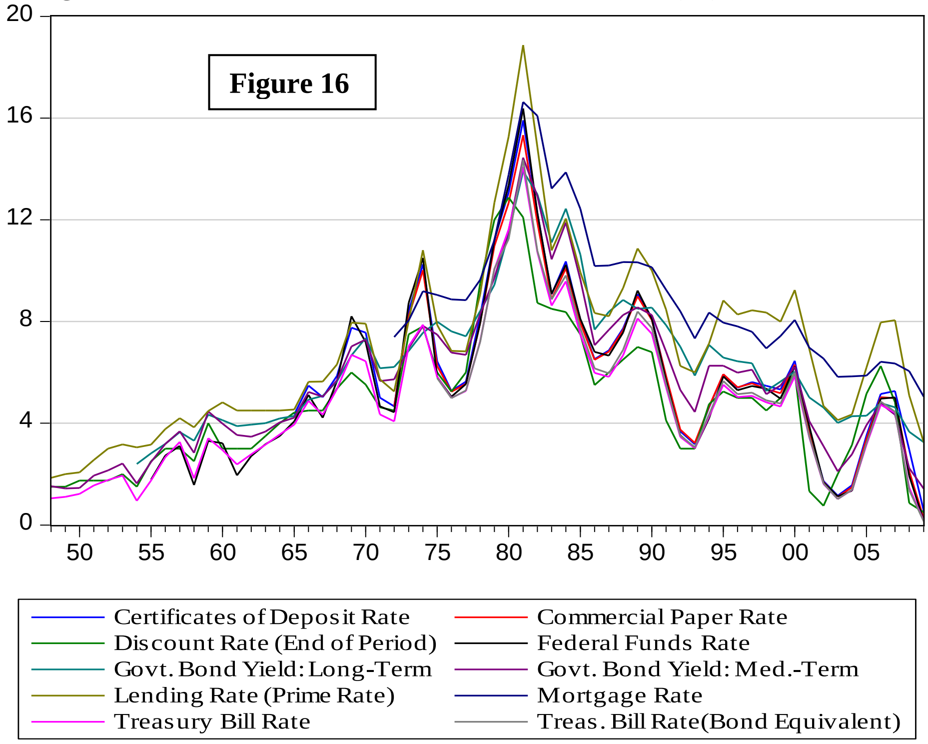


Table 1. Estimation results for ten estimated equations

Dependent Variable	Sample	obs.	$Y_t=ß_0 + ß_1.Yt_{-1} + ß_2.Y_{t-2}$				
			Y_t	$ß_0$	$ß_1$	$ß_2$	R^2
Certificates of Deposit Rate (secondary market-3 months)	1967-2009	43	CDR	1.412	1.173	-0.405	0.714
			t-Stat.	2.124	7.828	-2.675	
Commercial Paper Rate	1974-2009	36	CPR	1.092	1.208	-0.406	0.761
			t-Stat.	1.609	7.807	-2.564	
Discount Rate (End of Period)	1950-2009	60	DR	0.877	1.168	-0.349	0.774
			t-Stat.	2.387	9.376	-2.786	
Federal Funds Rate	1957-2009	53	FFR	1.159	1.121	-0.332	0.721
			t-Stat.	2.241	8.255	-2.445	
Lending Rate (Prime Rate)	1950-2009	60	LPR	1.193	1.195	-0.364	0.799
			t-Stat.	2.553	9.559	-2.962	
Treasury Bill Rate(Bond Equivalent-3 month)	1977-2009	33	TBRBE	0.920	1.212	-0.384	0.768
			t-Stat.	1.412	7.048	-2.153	
Mortgage Rate	1974-2009	36	MR	0.713	1.301	-0.386	0.874
			t-Stat.	1.140	7.983	-2.339	
Treasury Bill Rate	1950-2009	60	TBR	0.738	1.173	-0.330	0.792
			t-Stat.	2.126	9.257	-2.614	
Govt. Bond Yield: Long Term (10 year)	1956-2009	54	GBYLT	0.511	1.103	-0.180	0.880
			t-Stat.	1.491	7.973	-1.319	
Govt. Bond Yield: Medium Term (3 year)	1950-2009	60	GBYMT	0.539	1.127	-0.222	0.856
			t-Stat.	1.656	8.668	-1.718	

Table 2. Characteristic roots of the second order linear difference equations

Variables		$y_{t+2}=a.y_{t+1}+b.y_t+L$				Real	Imag.		Characteristic roots
		a	b	L	roots	c	d	D	x', x"
CDR	Certificates of Deposit Rate (secondary market-3 month)	-1.173	0.405	1.412	complex	0.586	0.865	1.045	0.586±0.865i
CPR	Commercial Paper Rate	-1.208	0.406	1.092	complex	0.604	0.878	1.065	0.603±0.877i
DR	Discount Rate (End of Period)	-1.168	0.349	0.877	complex	0.584	0.831	1.016	0.584±0.830i
FFR	Federal Funds Rate	-1.121	0.332	1.159	complex	0.561	0.804	0.980	0.560±0.803i
LPR	Lending Rate (Prime Rate)	-1.195	0.364	1.193	complex	0.598	0.849	1.039	0.597±0.849i
TBRBE	Treasury Bill Rate (Bond Equivalent-3 month)	-1.212	0.384	0.920	complex	0.606	0.867	1.058	0.606±0.866i
MR	Mortgage Rate	-1.301	0.386	0.713	real				0.843, 0.458
TBR	Treasury Bill Rate	-1.173	0.330	0.738	real				0.705, 0.469
GBYLT	Govt. Bond Yield: Long Term (10 year)	-1.103	0.180	0.511	real				0.903, 0.200
GBYMT	Govt. Bond Yield: Medium Term (3 year)	-1.127	0.222	0.539	real				0.872, 0.255

7 Conclusions

This paper examines the sources of business cycles. We tried to find the initiator of business cycles, which is in fact the dynamic structure of different interest rates. The study shows that there is important lag structure between deposit interest rates and loan interest rates. The observed lag structure actually forms a second order difference equation behavior in banking sector as source of fluctuations, which starts from money sector and expands to real economy. We tested this hypothesis on United States interest rates data for the long period of 1948-2009. The estimated results show that short-term interest rates are the source of fluctuations. The estimated dynamic equations for short-term interest rates had complex characteristic roots that let the equations be oscillatory. The long-term and medium-term interest rates equations had real characteristic roots and were not oscillatory.

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