Bijan Bidabad

General Monetary Equilibrium

Domestic, Foreign and International Monetary Equilibrium in Money and Commodity Markets

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Hu
121

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Table of Contents

Chapter One

Chapter Two

Chapter Three

Chapter Four

Chapter Five

Chapter Six

Hu

121

Preface

We are going to examine the bases of monetarism; hence, there are number of subjects that are discussed. Initially, we will try to investigate the philosophical bases of equilibrium in money and commodity markets, then try to reinterpret the "quantity theory of money" to express domestic monetary equilibrium. At this point, we will further deal with the important monetary variables as the velocity of circulation of money, rate of interest, supply and demand for money with emphasis on different demand motives. We next deal with external monetary equilibrium when foreign money exists and further explain how exchange rate and price levels are determined in an open economy. All these subjects are generalized to international monetary equilibrium. We will then go on to introduce some important monetary rules that exactly determine exchange rate at international level. The relation of interest rate and exchange rate is also determined. In the next section, we will reinterpret and develop the original Fisher's quantity theory of money by finding the quantitative link of total transactions and income in different integrated, disintegrated, and mixed production processes. Empirical investigations all confirm our model formulation. In the last section, we will try to define money logically and philosophically and the relation among transaction, output, intermediate input, aggregate supply and demand and their relations to money are explored.

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Chapter One

Introduction

There has always been and still there are many intellectual and philosophical controversies among monetary economists to give a concrete and well-defined foundation for monetary discussions. The philosophical debates that are milestones for developing theories are well treated but not sufficient in the realm of monetarism. Many monetary variables have not been thoroughly defined yet. Mainly, money itself does not have an exact definition to allow economists to refer it properly in a theoretical-empirical consistent framework. However, in different economies, it is concluded that the definition of money should be consistent with what can be used as money in that economy at a specific time. The degree of moneyness and public acceptability of different means as money are other problems that need to be recognized in each economy under consideration. In this regard, the particular content of money may vary from place to place and time to time. It is said that there is no sharp distinction between money and other assets in the real world transactions. Moreover, it is the asset holder's decision that determines what assets types are close (near) substitutes for one another and which are not. Emphasizes are made as to how one can define money as to sufficiently near substitute for other assets. As far as the definition of money is concerned, the most important issue is the identification and measurement of a stable aggregate demand function for money. Thus, in the context of this more general approach, the correct definition of money becomes a matter of an empirical analysis. Here, an important question arises; as to what is the theoretical definition of money that is consistent with its real world application. This is the point that turns the economist's attention to the demand motives for money or other kinds of assets that are included in the broader definition of money. On the other hand, money is defined based on the functions that it performs.

Illusions and indeterminacies in definition of money come from misspecification of functioning of money in the economy. On the other hand, money is defined commonly as anything that is generally acceptable as means of payment or means of final settlement of a debt. Thus, there still remains the problem of classification, of enumerating those items that are generally acceptable as means of payment. Acceptability is an attribute possessed by most assets - but in varying degrees. However, for this "concrete" definition of money as means of exchange or store of value, we cannot draw a borderline to distinguish money from other assets that may be physical or non- physical, tangible or non-tangible assets, goods, services and so on; because, all these assets can be used as medium of exchange and also as store of value. The "abstract" definition of money that thinks of money as a unit of account or measure of value also cannot identify the borderline of money and other assets. However, in a technologically changing world and continuous improvements in financial systems, drawing such a borderline is not an easy task. The following example shows the rejectability of both the concrete and abstract money definitions. The services that housewives offer their husbands, to look after children and manage home affairs are noticeable, but we do not add them up to value added calculations in national income accounts. However, this is because of problems and shortcomings of national income accounting, but, for these services, no money is allotted, but the wives' services are compensated by other types of services, by providing family expenditure and support that husbands offer their wives. In this particular case, the wives' services are exchanged by the husbands' services in absence of money. Therefore, money as means of exchange is not used and nothing is stored by money as value. Money is not also used to account the services exchanged or measuring the corresponding value. Therefore, concrete and abstract definitions do not include the phenomenon presented by this example. There are many examples that may be observed which do not satisfy these definitions and the borderline of money and other assets remains undefined. Economists discuss the functioning of money based on various monetary aggregates; each monetary aggregate has certain characteristics and compatible for specific issues. However, it is not a good remedy, but it prevails actually.

Now let us consider money as an economic commodity that has its own price and market consequences. If money works as a commodity, it must have a price. Its equilibrium price should be determined when its demand and supply meet each other in the market. Again another question arises: what is the "price of money" definition!? There are huge literatures about demand and supply of money, but in existing theories there is no coordination between the demand for money and supply in the market to determine the price of money. One of the goals of this book is to touch problem of the "price of money". The cornerstone of economic analysis is based on demand and supply analysis and the main variable in the demand and supply functions, is price. We always talk about demand and supply of money without referring to the "price of money". We cite the amount of quantity exchanged without reference to price of exchange. Some authors try to interpret the inverse of general price level of goods and services as price of money, but it is only an interpretation of a related variable and replacement of its inverse form instead of the lost "price of money variable. However, even this interpretation of "price of money" does not enter into demand and supply analysis of money theoretically. Some authors think of interest rate as money price. But, it is not really true, since when money is used for transactions, no interest is mentioned and when money is used for speculation interest will be the productivity of money - not the price of money. It is by this misunderstanding that we observe interest rate in theoretical and empirical formulation of demand for and supply of money. Ignorance of price in demand and supply functions is the same as neglecting the laws of demand and supply. However, we will deal with more subjects relating to this lost economic variable, but at this time, the reader may be confused between the two different terminologies of value and price. What we are talking about is price of money not value of money.

Before discussing the gradients of money, it is appropriate to check the money demand motives of transactions, speculative and precautionary which are the main categories in the literature (disregarding Keynes' business motive that is essentially of transaction type). Although some new theories of money demand, ignore this classification implicitly. Indeed, their derived demands actually can be decomposed in these categories. However, if we accept that definition of money comes from empirical stable demand for money function in the longer run - because of negligible net total speculative money demand - one may ignore the speculative motive part. Precautionary motive also can be neglected and regarded as portion of transaction demand for money. Thus, with these temporary simplifications, we can now concentrate on transaction motive that is the main base for the quantity theory of money. We will return to all of these motives.

It is noted here that demand for money theories (except a few) misinterpret the quantity theory of money. These theories actually do not accept that total value added in the economy finally is accumulated in the income (or total spending in the economy from the expenditure side of national income accounting)! For example, Keynesian view of demand for money correctly assigns transaction demand to total income, but incorrectly looks for speculative and precautionary demands elsewhere! Keynes adds these latter demands to the right hand side of equation [2.1] of the quantity theory of money. It should be asked that outside the total income (or expenditures) in the economy that can make added value that had not been included before in total income (or total value added). That is, when we talk about motives of demand for money we should look for them at the left hand side of the equation [2.1]. On the other hand, if one demands money for transaction, speculation or precautionary purposes, he is going to produce some added values (or income) and these values should be added and included in the total income to satisfy the definition of income. This is a rule in national income accounting. Transaction motive is to make income from transactions; speculative motive is to make income from speculations; precautionary motive is to make income from transactions and speculations both when unpredicted events occur. Specifically, speculative demand for money will produce some new added values that are part of income (or total value added) in the economy. Similarly, if one holds money to keep up himself from unpredicted events - preserving perfect foresight and rationality assumptions - he will have no idle money at the end of his own defined time period. At the end of the period, he has produced the related value added and has compensated himself from the occurred unpredicted events. This means that precautionary demand for money is formed in the income side and as before, nothing should be added as precautionary demand for money in the right hand side of equation [2.1] of the quantity theory of money. However, all these discussions are based upon this strong assumption that income is a scale variable for volume of transactions. Though we will show that this assumption is not accurate but to prevent confusion, we neglect this problem and continue the discussion. Let us consider the pioneer's definitional relations and our notification in interpretation of quantity theory of money:

Fisher's quantity theory:

> Money × Velocity = Price × Quantity of goods exchanged
> = Price × Volume of transactions
> = Value of transactions

Others' justification of Fisher's quantity theory:

> Money × Velocity = Price × Income

Keynes approach:

> Money × Velocity = Price × Income + Speculative money demand
> + Precautionary money demand
>
> Which implicitly assumes:
> Price × Income = Transaction demand for money.
>
> Note that in the Keynesian proposition, strangely, the different demands for money are located at the right hand side of the equation!

Our (initial) notification:

> (Transaction demand for money + Speculative demand for money) × Velocity
> = Price × Income
>
> Or, in another form:
>
> [Precautionary transaction demand for money + Precautionary speculative demand for money + Non-precautionary transaction demand for money + Non-precautionary speculative demand for money] × Velocity = Price × Income

At this point, we should note that we do not accept income as a straightforward scale variable for total volume of transactions. This point will be discussed more when we talk about the relation of income and volume of transactions in the following sections. However, if we accept "price × income" as a scale variable of total transaction, the right hand side does not change in the above specification but the left hand side includes all demand portions for money that is contradictory with Keynesian approach.

Quantity theory of money shows simultaneous equilibrium condition for money and commodity markets, and the importance of this theory is because of expressing this equilibrium. Keynesian theory actually distorts this equilibrium condition and implements it as a money demand equation incorrectly. When speculative and precautionary demands show up in the right hand side of quantity theory, "Price × Income" does not mean total value added in the economy and it means transaction demand for money. Moreover, "Price × Income" is not just transaction demand. Transaction demand is only one part of the total value added (or "Price × Income"). "Price × Income" is total of value added derived from transactions and speculations in both precautionary and non-precautionary situations. By these terms we mean, there exits different demands for money for the purposes of regular (non-precautionary) transactions, precautionary transactions, regular (non-precautionary) speculations. Non-precautionary transaction is the same regular transactions that occur just as Fisher explains. Precautionary transaction occurs for keeping the transactor up for unpredicted transactions. Non-precautionary speculation is that portion of speculations that occurs regularly as Keynes states and precautionary speculation is that remaining portion which occurs to make the speculator ready to benefit from the opportunities that are found suddenly.

Implicitly, the discussions in this book are based on the quantity theory of money. This theory (quantity theory) because of its strong philosophical background has proven its importance both in theoretical and empirical domains. We will go through this theory and try to develop it somehow to deplore its good equilibrium condition that can lead us to monetary equilibrium in national as well as international money and commodity markets.

Chapter Two

Domestic Monetary Equilibrium

2.1 Velocity of Circulation of Money, a Reinterpretation

In fact, Irving Fisher's equation of exchange defines the equilibrium condition of money and commodity markets. Although, economists label it as an accounting identity, and not a theory and emphasize on the interpretation of quantity theory by others who have given it a theoretical basis and also on the concept of money demand theory extracted from cash balance approach. However, we will explain more fundamentally its theoretical bases when we try to relate the volume of transactions to income by a one-to-one relation in the forthcoming sections. In his equation, Fisher proposed the quantity of money required to perform total transactions in the economy. However, other economists have changed the Fisher's terminology and they use total real expenditures (or income) as scale variable for volume of transactions. To stay away from further confusion, at this stage we will follow the revised approach and use the income variable instead of transactions volume variable. In the forthcoming sections, we will return to this important point of misunderstanding. Let us see the following important quantity theory equation in its revised form. We used the term "revised form" because; Fisher used total nominal transaction at the right hand side of the following equation instead of nominal income. However, we will come back to the original form later.

$MV = Py = Y$ [2.1]

where;

M: Supply of (or demand for) money.

V: Number of times an average unit of money changes hands (velocity of circulation of money).

P: Price level.

y: Total quantity of goods and services sold.

Y: Nominal income or aggregate expenditure.

In this section, we will just try to manipulate the "V" variable definition for other purposes that come later. In equation [2.1] assume that the nominal GNP (Y) is constant. For different values of (Y) we can sketch the corresponding curves in the M-V plane as in figure 2.1.

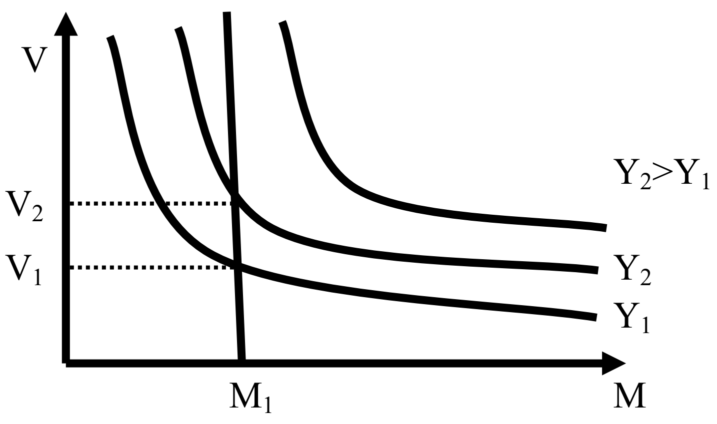


Figure 2.1

Now, without any further explanation, assume that "V" is a "price index for money" until we explain its relevancy. Moreover, assume "M" calls for demand for money and the hyperbolic curves are money demand curves. These hyperbolas are simply derived by writing "V" as a function of "M" from equation [2.1], that is V=Y/M. At given value of total nominal income say "Y_1"; for different values of "M" we have different values for "V". On the other hand, for any quantity of money demanded we have a specific value as the "price of money" or what we call as velocity of circulation of money. Thus, when price of commodities or quantity of output is high, with a given stock of money (M_1) the price of money should be higher to decrease the money demand to preserve the equilibrium. On the other hand, velocity of circulation should be faster to facilitate the nominal transactions. In the following discussion, we are going to prove that velocity of circulation of money is somehow the price of money and in this regard, the depicted curves on figure 2.1 are money demand curves.

In a similar fashion - that prices of other commodities are defined - we can redefine velocity of circulation of money as the price of money as follows. Always, price of a commodity is the value of one unit of the corresponding commodity in the market at the time of transaction. Similarly, we define the price of money as the value of one unit of money in the market (but) at the end of the specified period, that money has been circulated (not at the moment of time). Note, when we define price of commodity we emphasize on a specific moment of time and when we define price of money we refer to a specific period. Total value of purchased or sold commodity is equal to price of commodity multiplied by their exchanged quantity or "Py" in aggregate term. The total value of money exchanged ("purchased or sold") is equal to the quantity of money multiplied by the times that it circulates in the economy, which is "MV" in aggregate term. In this regard, velocity acts as the price of money. For more clarifications, now concentrate on the following relations sequentially and note to differences between value of money and price of money in the reasoning. At this time, we will just talk about money, so the reader should forget the relation between value of money and price of commodity and purchasing power of money context.

Price of one unit of money at a moment of time =
Value of one unit of money at a moment of time =
Quantity of one unit of money at a moment of time.

Price of one unit of money in a period of time =
Value of one unit of money in a period of time =
Quantity of one unit of money in the 1st circulation +
Quantity of one unit of money in the 2nd circulation +
Quantity of one unit of money in the 3rd circulation +
....+
....+
....+
Quantity of one unit of money in the Vth circulation.

Price of one unit of money in a period of time =
Value of one unit of money in a period of time =
(**Quantity** of one unit of money in circulation) × V.

Price of one unit of money in a period of time =
Value of one unit of money in a period of time =
Velocity of circulation of money.

To clarify this, let us see the following example. One sells or purchases one unit of a commodity by making a payment value equal to its price. Thus, purchased or sold value of one unit of commodity is equal to the price of that commodity. On the other hand, the price is the nominal exchanged value of one unit of commodity. Similarly, one unit of money is exchanged (or circulated) V times in a specified period. Therefore, the exchanged value of one unit of money is equal to the times that it is exchanged or circulated. Therefore, the payment to purchase one unit of money (similar to commodity) is equal to the quantity of money multiplied by the times of circulations.

Now assume that precautionary and speculative demands for money do not exist. Therefore, the above definition for price of money is valid in the transaction domain. To sum up according to the above reasoning, we can define the "transaction price of money" (price of money when money is used for transaction) as the times that average unit of currency circulates in economy to facilitate transaction. However, this is just a reinterpretation of velocity and it does not actually change the conclusions we will make in forthcoming discussions about different monetary debates. On the other hand "velocity" or "price of money" whatever we call it is a variable that establishes the equality (or equilibrium) of nominal transaction with nominal money payment for that transaction.

2.2 Rate of Interest, a Reinterpretation

In this section, we try again to define the "speculative price of money" (price of money when money is used for speculation). Similar to the previous section, the speculative price of one unit of money is the value of the same money unit plus the amount of returns that speculator receives at the end of the specified period all multiplied by the times of circulations. So, if we denote rate of interest by "i", the amount that speculator will receive at the end of period by investing one unit of money will be "1+i". But this is the case when money is circulated only once in the corresponding period. That is, when money goes in the hands of other speculators, they also receive "1+i" units of money at end of period (requires a given interest rate). The times that speculators invest their money is equal to the velocity of circulation of money for this purpose. Total receipts for one unit of money that is invested in speculation will be equal to V(1+i) where "V" denotes the velocity of circulation of money for speculative purposes. Thus the total received value of V(1+i) for one unit of money can be denoted as the "speculative price of money". This becomes clearer by the following reasoning.

Price of one unit of money in a period of time with interest rate i =
Value of one unit of money in a period of time with interest rate i =
Quantity of one unit of money + amount of return (i) at the end of period in the 1st circulation +
Quantity of one unit of money + amount of return (i) at the end of period in the 2nd circulation +
Quantity of one unit of money + amount of return (i) at the end of period in the 3rd circulation +
.... +
.... +
.... +
Quantity of one unit of money + amount of return (i) at the end of period in the Vth circulation.

Price of one unit of money in a period of time with interest rate i =
Value of one unit of money in a period of time with interest rate i =
(**Quantity** of one unit of money in circulation + amount of return (i) at the end of period in the 1st circulation) × V

Price of one unit of money in a period of time with interest rate i =
Value of one unit of money in a period of time with interest rate i =
(**one + interest rate**) × Velocity of circulation of money

Total value of money demanded for speculative purposes will be the amount of money allocated for investment in this activity multiplied by speculative price of money V(1+i).

2.3 Risk Rate, a Reinterpretation

Precautionary demand for money is the amount that one holds to tackle unanticipated events. If he could foresee the future, he would hold that amount of money to compensate the total nominal value of unanticipated events. Keynes' precautionary motive restatement of Cambridge security motive is "to provide for contingencies requiring unpredicted expenditure and for unforeseen opportunities of advantageous purchases and also to hold an asset of which the value is fixed in terms of money to meet a subsequent liability fixed in terms of money". In this regard if we denote the risk rate by "r", total value of unpredicted expenditure will be equal to "rPy". The remaining value of (1-r)Py will be related to "predicted" expenditures. Now decompose the equation [2.1] as follows:

$$(1-r)MV + rMV = (1-r)Py + rPy \qquad [2.2]$$

In the left side of the above equation, total value of money exchanges has been decomposed to two parts. (1-r)MV is the portion of total money exchanges related to "predicted" transactions (1-r)Py and another part "rMV" is that portion of total money exchanges which is related to "unpredicted" transactions "rPy". In this regard, "rMV" is redefined as "precautionary money demand". When the risk rate is equal to zero, it means that all of the transactions are of "predicted" form and no money is held for precautionary purposes. Thus, the risk rate just allocates money to precautionary and non-precautionary demands portions and it does not change value of money as we had before in transaction and speculation discussions. So, when it does not change value of one unit of money it also does not change the price of money.

Now, let us go further and express the "price of money held for precautionary purposes". The expression "rM" is the amount of money that is held for precautionary purposes, so, the "V" variable in the expression "rMV" will be exactly the price of money held for precautionary purposes. That is velocity of circulation of money again becomes price of money, since risk rate does not change the value of money.

2.4 Velocity and Different Motives

Up to this juncture, we had assumed that velocity of circulation of money might not change in different motives of holding money. It can be said that when money is used for speculative purposes it circulates very faster than those moneys that are held for transactions or precautionary purposes. Similarly, transaction money has higher velocity than money held for precautionary aim. However, if we denote V_S, V_T and V_P for velocities of circulation of money for speculative, transaction and precautionary purposes respectively, the following inequality should exist:

$$V_S > V_T > V_P$$

2.5 Money Supply and Demand

Now, return to the quantity theory of money given by equation [2.1]. Let decompose the total money demand "M" to transaction, speculative and precautionary demands and denote them by M_T, M_S and M_P respectively. That is;

$$M = M_T + M_S + M_P \qquad [2.3]$$

In order to equate total money value to total value added in the economy we should multiply each portion of money demands to the corresponding money prices. Denote prices of transactions, speculative and precautionary moneys to P_T, P_S and P_P respectively. Following the discussions made before, we have;

$P_T = V_T$ (transaction price of money)

$P_S = V_S(1+i)$ (speculation price of money)

$P_P = V_P$ (precautionary price of money)

where V_T, V_S and V_P are velocities of circulation of money when money is used for transaction, speculation and precautionary purposes respectively.

The total money required to circulate total value added (produced by both precautionary and non-precautionary transactions and speculations) in the economy will be:

$$M_T P_T + M_S P_S = M_T V_T + M_S V_S(1+i) \qquad [2.4]$$

This equation is derived by substituting the above identities into left hand side of [2.4]. By quantity theory of money we may write,

$$M_T V_T + M_S V_S(1+i) = Py \qquad [2.5]$$

But $M_P V_P$ (precautionary demand for money multiplied by its velocity) is the portion of "unpredicted" (precautionary) transactions and speculations in total MV. On the other hand the "unpredicted" portion is equal to $r(M_T P_T + M_S P_S)$ where r is the risk rate. Thus, we can deduce precautionary portion to;

$$M_P V_P = r[M_T V_T + M_S V_S(1+i)] \qquad [2.6]$$

Remaining "predicted" (non-precautionary) portion will be equal to $(1-r)[M_T V_T + M_S V_S(1+i)]$. Therefore, the exchange equation can be written as;

$$(1-r)[M_T V_T + M_S V_S(1+i)] + r[M_T V_T + M_S V_S(1+i)] = Py \qquad [2.7]$$

The expression $r[M_T V_T + M_S V_S(1+i)]$ is equal to precautionary portion of exchange values. Thus, we have;

$$(1-r)M_T V_T + (1-r)(1+i)M_S V_S + P_P M_P = Py \qquad [2.8]$$

or,

$$(1-r)M_T V_T + (1-r)(1+i)M_S V_S + V_P M_P = Py \qquad [2.9]$$

where,

$$V_P M_P = r[M_T V_T + M_S V_S(1+i)] \qquad [2.10]$$

The equation [2.9] is again expressing the quantity theory of money with the following gradients:

$(1-r)M_T V_T$	Portion of required money value for predicted transactions.
$(1-r)(1+i)M_S V_S$	Portion of required money value for predicted speculations.
$P_P M_P$	Portion of required money value for total unpredicted and predicted transactions and speculations that is equal to $r[M_T V_T + M_S V_S(1+i)]$.

For the sake of simplicity, assume that all velocities are equal;

$$V = V_T = V_S = V_P \qquad [2.11]$$

This assumption is highly near to the spirit of quantity theory. Because Fisher defines velocity as the times that "an average" unit of money changes hands. In this regard when money goes from hands of speculator to the hands of transactor or precautionary money holder, we cannot distinguish the purposes of holding money and the phrase "an average" helps us to cover all the cases of money demands.

The equation of exchange will be:

$$V\{(1-r)[M_T+(1+i)M_S]+M_P\} = Py \quad [2.12]$$

where;

$$M_P = r[M_T+(1+i)M_S] \quad [2.13]$$

Precautionary portion of hoarded money, as cited before, is just a "r" fraction of total money "M". Therefore, we may write the equation of exchange as:

$$V[M_T+(1+i)M_S] = Py \quad [2.14]$$

This equation is derived by substituting [2.13] in [2.12]. Compare to quantity theory notation, necessarily we have the following equation for the total amount of money "M" as;

$$M = M_T + (1+i)M_S \quad [2.15]$$

Note that the precautionary portion is included in definition of money gradients in [2.15]. We may decompose [2.15] as;

$$M = (1-r)[M_T+(1+i)M_S] + r[M_T+(1+i)M_S] \quad [2.16]$$

In this case the equilibrium condition will be;

$$V\{(1-r)[M_T+(1+i)M_S] + r[M_T+(1+i)M_S]\} = Py \quad [2.17]$$

In connection with the previous argument given by [2.15], the results from figure 2.2 would be of great interest. In figure 2.2, "M" is depicted versus interest rate "i". The sloped line is the supply of money at different interest rates. The expression $(1+i)M_S$ in equation [2.15] may be decomposed in two parts of autonomous speculative and induced speculative demands. Name "M_S" as autonomous and "iM_S" as induced portions of speculative money. When i=0, we have $M=M_T+M_S$, that is no induced speculation exists and interest rate has no effect on total supply of money. When i>0, total money supply will increase by amount of induced speculative money "iM_S". We aim to clarify that at any interest rate "i_0" (as in figure 2.2) we have a corresponding money supply equal to "M_0". The predecessor theories assume that the money supply is vertical at M_T+M_S in figure 2.2. The recent theories that observe endogenous money supply, consider a positive relationship between money and interest rate. However, though they do not follow such a confusing analysis and they do not specify the theoretically well-organized characteristics and functional form of this relationship as it is observed in figure 2.2.

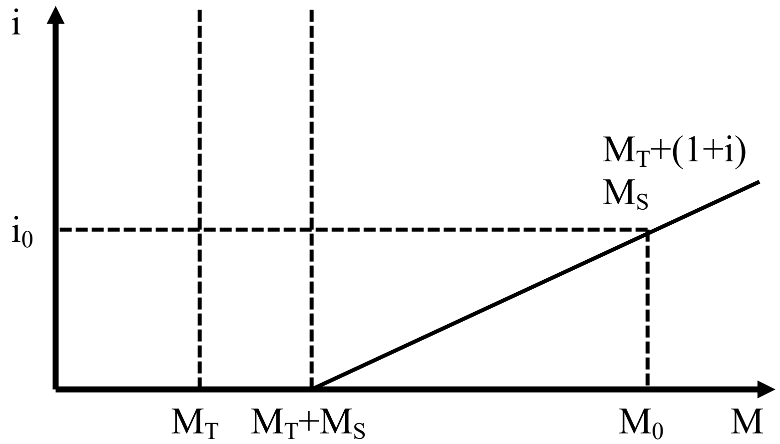


Figure 2.2

Now, let us discuss the demand for money from equation [2.14]. Total value added in the economy (Py) may be decomposed into three parts of value added derived from transactions and autonomous and induced speculations that are denoted by $(Py)_T$, $(Py)_A$ and $(Py)_I$ respectively. This implies that;

$$Py = (Py)_T + (Py)_A + (Py)_I = Y_T + Y_A + Y_I = Y \qquad [2.18]$$

Where all T, A and I subscripts are related to transaction, autonomous speculation and induced speculative variables. The transaction and autonomous and induced speculation demands for and supplies of money are depicted in figures 2.3, 2.4 and 2.5 respectively. In these figures, D and S denote demand and supply in the corresponding markets that are received from the following decomposition;

$$
\begin{array}{lclcl}
VM_T & = & (Py)_T & = & Y_T \\
+\,VM_A & = & +\,(Py)_A & = & +\,Y_A \\
+\,ViM_I & = & +\,(Py)_I & = & +\,Y_I \\
\hline
=V(M_T+M_A+iM_I) & = & Py & = & Y
\end{array}
$$

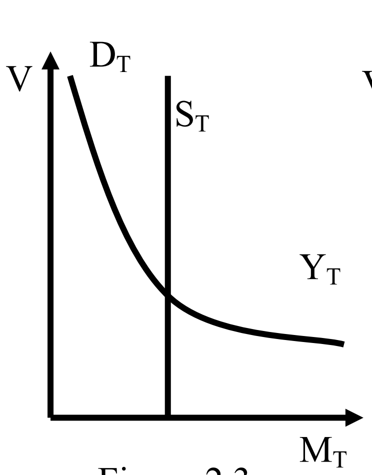


Figure 2.3

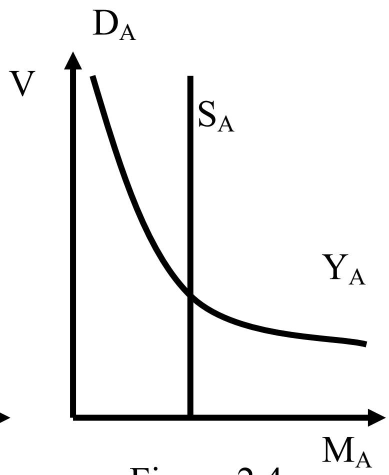


Figure 2.4

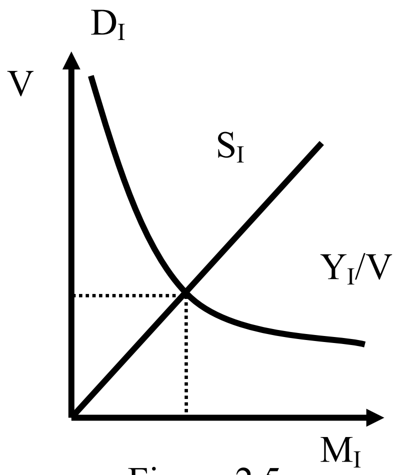


Figure 2.5

The demand functions depicted in these figures are for total unpredicted and predicted portions. As it was discussed earlier unpredicted (precautionary) and predicted (non-precautionary) demands for money are convex combinations with "r" and "1-r". These demand’s portions depicted by figures 3 through 5 can be displayed separately as shown by figures 2.6, 2.7 and 2.8.

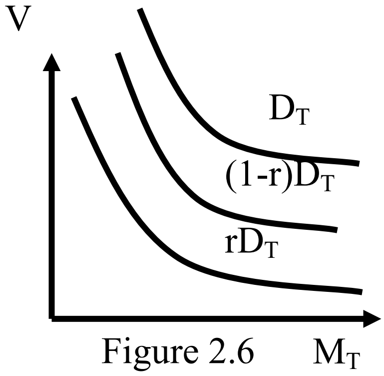


Figure 2.6

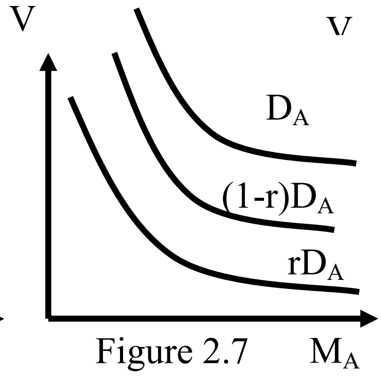


Figure 2.7

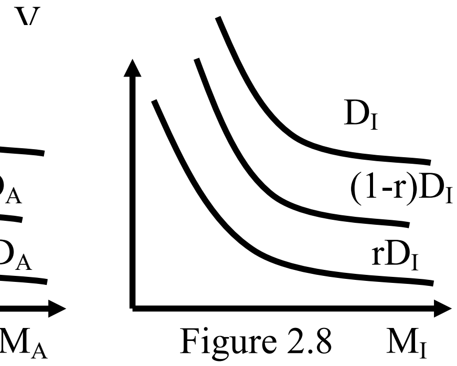


Figure 2.8

Up to this juncture, we have derived the supply and demand for money in the revised frame of quantity theory. To sum up, we may look at equation [2.14] as the equilibrium condition of money and commodity markets; or it may be checked as money demand equation. Relation [2.15] is money supply equation. The distribution of precautionary and non-precautionary portions of money supply is shown by equation [2.16]. The demand for these portions of money is depicted by [2.17].

2.6 Empirical Analysis

In this section, we intend to test two important propositions that we raised through the above sections. The first proposition to be tested is money demand equation presented by the frame of figure 2.1. That is we are going to test that: "is there any hyperbolic relationship between money stock and price of money (or velocity of circulation of money)"? The second proposition to be tested is the equilibrium condition of money and commodity markets in our revised form of quantity theory given by equation [2.14]. That is we intend to test that does equilibrium condition [2.14] prevail?. To perform the tests we used the data gathered by Friedman and Schwartz (1982, pp. 121-129) for the United States of America. Definitions of used symbols in these tests are as follows:

$i_t^{(1)}$ = Short term commercial paper rate (annual percentage).

$i_t^{(2)}$ = Short term call money rate (annual percentage).

$i_t^{(3)}$ = Long term yields on high-grade corporate bonds (annual percentage).

$i_t^{(4)}$ = Long term yields on high-grade industrial bonds (annual percentage).

$i_t^{(5)} = [i_t^{(1)}+i_t^{(2)}]/2$.

$i_t^{(6)} = [i_t^{(3)}+i_t^{(4)}]/2$.

$i_t^{(7)} = [i_t^{(5)}+i_t^{(6)}]/2$.

P_t = GNP implicit price deflator, 1929=100.

y_t = Real Income (billion 1929 \$).

M_t = Money stock (billion \$).

V_t = Velocity of circulation of money, $(P_t y_t/M_t)$.

t = Time subscript.

ρ = Autoregressive parameter.

2.6.1 Test of Hyperbolic Shape of Money Demand

Our null hypothesis here is: "there exists a strong hyperbolic functional relationship between money demand as a whole and price of money or velocity of circulation of money". This test actually is performed to investigate the accuracy of figure 2.1 and corresponding justifications. If the null hypothesis is true, we should have a significant dominant functional relationship between money and its velocity in the long run as;

$$M_t = \text{ß}_1 \frac{1}{V_t} + u_t \qquad [2.19]$$

These points had simply derived from the quantity theory of money given by [2.1].

To check the null hypothesis doubly, we may add an intercept to equation [2.19] to conclude that if the estimated intercept were not significant then the equation [2.19] will more significantly explains the truism of the null hypothesis. Thus, regression equations [2.19] and following [2.20] are chosen for calculations.

$$M_t = ß_0 + ß_1 \frac{1}{V_t} + u_t \qquad [2.20]$$

Results of applying Cochrane-Orcutt procedure for removing first order serial correlation in least squares method applied to the data for the period of 1870-1975 are summarized in table 2.1. By table 2.1, we can accept the null hypothesis and the results confirm our theoretical discussions about the shape of money demand against price of money.

Table 2.1

Eq. No.	Dep. Var.	Ind. Var.	$ß_0$ ($S_{ß0}$^)	$ß_1$ ($S_{ß1}$^)	ρ (Sρ)	Time range	R^2	Durbin-Watson
2.19	M_t	$1/V_t$	------- -------	26.230 (9.743)	1.0791 (0.0035)	1870-1975	0.9989	0.8200
2.20	M_t	$1/V_t$	*3.237 (8.110)	23.648 (11.67)	1.0796 (0.0037)	1870-1975	0.9980	0.8107

* Insignificant

2.6.2 Test of the Revised Form of Quantity Theory

Our null hypothesis in this section is: "the revised form of the quantity theory of money given by equation [2.14] is confirmed". To test the equation [2.14], we tried to write it as the following equation, which can be simply derived from [2.14];

$$i = -\frac{\frac{M_T+M_S}{P}}{\frac{M_S}{P}} + \frac{1}{\frac{M_S}{P}}\left(\frac{y}{V}\right) \qquad [2.21]$$

Denote i and y/V as dependent and independent variables in the following specification of our regression.

$$i_t = ß_0 + ß_1 \left(\frac{y_t}{V_t}\right) + u_t \qquad [2.22]$$

Obviously, in this specified model (comparing [2.21] to [2.22]) we have;

$$ß_0 = -\frac{\frac{M_T+M_S}{P}}{\frac{M_S}{P}} = -\frac{\frac{M_T}{P}}{\frac{M_S}{P}} - 1; \quad ß_1 = \frac{1}{\frac{M_S}{P}} \qquad [2.23]$$

As it is obvious from [2.23] the estimates of the coefficients $ß_0$ and $ß_1$ should have the following properties;

$$ß_0 < -1; \quad 0 < ß_1 < 1 \qquad [2.24]$$

However, after estimating [2.22] we can accept the null hypothesis if the conditions given by [2.24] hold at suitable confidence interval. To examine this hypothesis, we used seven interest rates including short and long terms both. The selected rates were cited before. The results of calculated regressions are depicted in table 2.2. All the reported regressions confirm our hypothesis and model formulation regarding all of the calculated statistics and the important conditions of [2.24]. However, we could not find even one case that rejects our hypothesis of the proposed (revised) version of the quantity theory of money. Therefore, we can actually accept the equation [2.14] as the equilibrium condition of money and commodity markets.

Table 2.2

Eq. No.	Dep. Var.	Ind. Var.	β_0 ($S_{\beta 0}$^)	β_1 ($S_{\beta 1}$^)	ρ ($S\rho$)	Time range	R^2	Durbin-Watson
1	$I_t^{(1)}$	y_t/V_t	-5.170 (1.142)	0.068 (0.008)	0.2265 (0.1899)		0.8029	1.6799
2	$I_t^{(2)}$	y_t/V_t	-4.420 (1.589)	0.067 (0.011)	0.4683 (0.1665)	1945-1975	0.8264	1.5908
3	$I_t^{(3)}$	y_t/V_t	-3.060 (0.810)	0.057 (0.005)	0.5481 (0.1599)	1945-1975	0.9426	1.4223
4	$I_t^{(4)}$	y_t/V_t	-3.521 (0.740)	0.061 (0.005)	0.5261 (0.1581)	1945-1975	0.9526	1.3573
5	$I_t^{(5)}$	y_t/V_t	-4.825 (1.313)	0.068 (0.009)	0.3496 (0.1789)	1945-1975	0.8177	1.6235
6	$I_t^{(6)}$	y_t/V_t	-3.280 (0.771)	0.059 (0.005)	0.5433 (0.1582)	1945-1975	0.9495	1.3604
7	$I_t^{(7)}$	y_t/V_t	-4.047 (0.939)	0.063 (0.006)	0.4239 (0.1708)	1945-1975	0.9046	1.5726

2.7 Supply of Money

In this section, we try to develop a model of money supply close to existing literature with a major difference assumption of finite circulation of money. By this assumption, mechanism of monetary expansion is in a form that any increase in the reserves of commercial banks will increase the money supply by inverse of reserve requirement ratio against demand deposits. The inverse of this reserve requirement ratio comes from an infinite geometric series. This infinite series actually presumes that infinite times money circulates through the hands of public and commercial banks. We are going to restrict this times of circulation of money to a real finite number as the number of times a unit of money changes hands.

To clarify the proposition, assume fully invested banks, no transfer of demand deposits to time or saving deposits and no currency holding by public and suppose initially that, public deposits his checks equal to amount of "R" units in his checking account in commercial banks. This creates "R" units of liabilities for the banks, the claim on the bank by the depositor, and also "R" units in assets for the bank, the claim on the central bank. If there is a z.100 percent reserve requirement ratio, commercial banks can loan (1-z)R units and must retain zR as reserves. The borrower of (1-z)R presumably spends it, transferring the (1-z)R again to commercial banks, which can in turn loan out $(1-z)^2R$ units. This amount is transferred to commercial banks and process continues. As a result, the demand deposits segment (DD) of money supply (M) created by R units of reserve will be equal to:

$$DD = R+(1-z)R+(1-z)^2R+... = R[1+(1-z)+(1-z)^2+...+(1-z)^{\infty}] \qquad [2.25]$$

Sum of the geometric series inside the brackets will be equal to 1/z. Thus, total demand deposits will be equal to,

$$DD = R/z \qquad [2.26]$$

Now, let's scrutinize this equation. Suppose that central bank decides to reduce the reserve requirement ratio near or equal to zero. If this is the case, demand deposits and thus money supply "M" tends to infinity and when z=0, M will be infinity. That is,

$$\lim_{z \to 0} DD = \lim_{z \to 0} (R/z) = \infty \qquad [2.27]$$

But this must not be actually the case. Money supply will never be infinity even reserve requirement ratio is zero. This problem occurs because the geometric series in

[2.25] has infinite terms. Number of terms in [2.25] actually expresses the number of times that money is transferred to commercial banks; which is not actually infinite. Thus, if reserve requirement ratio is even zero total money created should be equal to initial reserve multiplied by the times money is loaned out by commercial banks. On the other hand, total demand deposits should be equal to initial reserve multiplied by the times of circulation of money. This means that if the times of circulation of money "w" is an integer, money supply creation stops at w^{th} round of money circulation. That is we have following "w" terms for "w" rounds of money circulation. To clarify this, note to the mechanism of money creation given by the following table 2.3;

Table 2.3

	New deposits	Required reserve	Loan and investment	Total reserves
w	DD	RR	ER	RR+ER
1	R	zR	(1-z)R	R
2	(1-z)R	z(1-z)R	$(1-z)^2R$	(1-z)R
3	$(1-z)^2R$	$z(1-z)^2R$	$1-z)^3R$	$(1-z)^2R$
4	$(1-z)^3R$	$z(1-z)^3R$	$(1-z)^4R$	$(1-z)^3R$
. . .	. . .	. . .	. . .	. . .
w	$(1-z)^{w-1}R$	$z(1-z)^{w-1}R$	$(1-z)^wR$	$(1-z)^{w-1}R$

Table 2.3 shows that in the first round of money circulation, initial reserve is equal to "R", the amount of reserve requirement is equal to zR and demand deposit created is equal to "R". Excess reserve (ER) will be equal to (1-z)R which again goes through the banking system and becomes demand deposit at banks at second round of circulation. RR+ER in each round of circulation is equal to total reserve or DD at the

same round. Thus, RR and ER show the distribution of reserves into reserve requirement and excess reserve. The amount of excess reserve again becomes reserve of banking system at the next round. This process continues until we reach to the w^{th} round of money circulation. Total amount and distribution of reserve requirement and deposits for the "w" rounds of circulation will be equal to the following equations;

$$DD = R[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}] \quad [2.28]$$

$$RR = zR[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}] \quad [2.29]$$

$$ER = (1-z)R[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}] \quad [2.30]$$

$$RR + ER = R[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}] \quad [2.31]$$

When a non-zero reserve requirement ratio exists by using the sum of the geometric series, [2.28], through [2.31] will be reduced to the following expressions;

$$DD = (R/z)[1-(1-z)^w] \qquad z \neq 0 \quad [2.32]$$

$$RR = R[1-(1-z)^w] \qquad z \neq 0 \quad [2.33]$$

$$ER = [(1-z)/z]R[1-(1-z)^w] \qquad z \neq 0 \quad [2.34]$$

$$RR + ER = (R/z)[1-(1-z)^w] \qquad z \neq 0 \quad [2.35]$$

By replacement of z=0 in the [2.28] through [2.31] we will find the following expressions for the case of zero reserve requirement ratio;

$$DD = Rw \qquad z = 0 \quad [2.36]$$

$$RR = 0 \qquad z = 0 \quad [2.37]$$

$$ER = Rw \qquad z = 0 \quad [2.38]$$

$$RR + ER = Rw \qquad z = 0 \quad [2.39]$$

Thus if we suppose the reserve is exogenously determined by the central banks, the amount of money supplied by monetary expansion mechanism is equal to [2.32] and [2.36] for different cases of non-zero and zero reserve requirement ratios. But, in the definition of money supply we should add currency in hands of public in order to define narrow money "M1" as follows;

$$M1 = CP + (R/z)[1-(1-z)^{w}] \qquad [2.40]$$

Where, CP is currency in hands of public.

However, there is a problem with above formulation. It is assumed that "w" is an integer variable. This means we have presumed that all of reserves exhaust in each round of money circulation. That is, in the first round all excess reserves become demand deposits. Then these deposits are transformed to banks to perform second round of excess reserves, and so on. But, in practice, excess reserves do not exhaust all at once in a lump sum. On the other hand, there is two contemporaneous streams of transitions of demand deposit to reserve and reserve to demand deposit. This changes the nature of the times of circulation of money from an integer to a real type variable. In this case, our above formulation changes slightly. The sum of the geometric series of [2.28] through [2.31] should be transformed to integrals of compound geometric series. Assume that the amount of demand deposits created at the t^{th} round of circulation is equal to DD_t given by the following expression,

$$DD_t = R(1-z)^{t} \qquad [2.41]$$

If during the t^{th} round we have "k" times of transitions of reserves to deposits we may write [2.41] as;

$$DD_t = R(1-z/k)^{tk} \qquad [2.42]$$

Now assume that m=k/z, thus [2.42] can be written as;

$$DD_t = R[(1-1/m)^{-m}]^{-zt} \qquad [2.43]$$

If "k" tends to infinity "m" tends to infinity too, then [2.43] will be equal to the following limit expression;

$$DD_t = \lim_{m \to \infty} R[(1-1/m)^{-m}]^{-zt} = Re^{-zt} \qquad [2.44]$$

where "e" is natural base of logarithm. Total demand deposits created during zero to w^{th} rounds of money circulation will be equal to the following definite integral;

$$DD = \int_0^w Re^{-zt}\,dt = (R/z)(1-e^{-zw}) \quad z \neq 0 \qquad [2.45]$$

Similar to [2.29] through [2.31], we have the following expressions for continuous case:

$$RR = R(1-e^{-zw}) \qquad z \neq 0 \qquad [2.46]$$

$$ER = [(1-z)/z]R(1-e^{-zw}) \qquad z \neq 0 \qquad [2.47]$$

$$RR + ER = R(1-e^{-zw}) \qquad z \neq 0 \qquad [2.48]$$

The equations [2.36] through [2.39] again hold for the case of z=0. Similar to equation [2.40] we may define narrow money as;

$$M1 = CP + (R/z)(1-e^{-zw}) = CP + DD \qquad [2.49]$$

Expressions [2.40] and [2.49] have very near values for higher times of circulation and lower reserve requirement ratios.

2.7.1 Interest Deposit

In this section, we try to bridge the discussion of money supply of previous section to argument raised by equations [2.14] and [2.15]. By definition, the term "interest deposit" means those payments due to the interest of all interest-bearing deposits. On the other hand, interest deposit is the amount of interest payments to interest-bearing deposits holders in form of banking deposits. By this term, we do not introduce any new banking deposit. We are just going to bring all interest payments of banking system to deposit holders into account, because all interest payment are done by some kinds of banking deposits instruments.

Table 2.4

	Demand deposits + time deposits	Demand deposits	Time deposits	Reserve requirement
w	DD+TD	DD	TD	RR
1	R	(1-d)R	dR	zR
2	(1-z)R	(1-d)(1-z)R	d(1-z)R	z(1-z)R
3	$(1-z)^2R$	$(1-d)(1-z)^2R$	$d(1-z)^2R$	$z(1-z)^2R$
4	$(1-z)^3R$	$(1-d)(1-z)^3R$	$d(1-z)^3R$	$z(1-z)^3R$
. . .	. . .	. . .	. . .	. . .
w	$(1-z)^{w-1}R$	$(1-d)(1-z)^{w-1}R$	$d(1-z)^{w-1}R$	$z(1-z)^{w-1}R$

Table 2.4 continued

	Loan and investment excess reserves	Interest deposits	Total money supply (demand deposit + time deposit + interest deposits)
w	ER	ID= i TD	DD+TD+ID
1	(1-z)R	dRi	R(1+di)
2	$(1-z)^2R$	d(1-z)Ri	(1-z)R(1+di)
3	$(1-z)^3R$	$d(1-z)^2Ri$	$(1-z)^2R(1+di)$
4	$(1-z)^4R$	$d(1-z)^3Ri$	$(1-z)^3R(1+di)$
. . .	. . .	. . .	. . .
w	$(1-z)^wR$	$d(1-z)^{w-1}Ri$	$(1-z)^{w-1}R(1+di)$

To show how interest deposit is created through monetary expansion mechanism, note the table 2.4. The initial reserve "R" goes to the hands of public for the first time of circulation. Public deposits an amount equal to "(1-d)R" in his checking account and an amount equal to "dR" in his saving account, where $0 \leq d \leq 1$. Accordingly, this means that "d" is the ratio of time deposits to sum of demand and time deposits, as;

$$d = TD/(DD+TD) \qquad [2.50]$$

By the word time deposits, we mean all existing interest bearing deposits and the term demand deposit refers to class of all non-interest bearing deposits. Assume that reserve requirement ratio is "z" for all types of deposits. Bank reserve requirement "RR" will be equal to "zR" and amount of excess reserve will be "(1-z)R". For this round of circulation bank should pay some interest deposit equal to "iTD" to depositor at the end of period as interest payment. Therefore, an amount equal to "dRi" as interest deposit is also created. Total expansion of money supply for the first round will be equal to "M~" as some of all demand, time and interest deposits. For the second time of circulation, excess reserve of the first round is loaned out by the bank and again an amount of "(1-z)R" equal to excess reserve of the first round is created as demand and time deposits. The process continues until we reach the w^{th} round of circulation. Total amount of deposits, reserve requirement, excess reserve, interest deposit and money supply for "w" times circulations will be equal to following series;

$$DD + TD = R[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}] \quad [2.51]$$

$$DD = (1-d)R[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}] \quad [2.52]$$

$$TD = dR[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}] \quad [2.53]$$

$$RR = zR[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}] \quad [2.54]$$

$$ER = (1-z)R[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}] \quad [2.55]$$

$$ID = diR[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}] \quad [2.56]$$

$$M\sim = (1+di)R[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}] \quad [2.57]$$

Alternatively, we may write the above series as following formulas;

$$DD + TD = (R/z)[1-(1-z)^w] \quad [2.58]$$

$$DD = (1-d)(R/z)[1-(1-z)^w] \quad [2.59]$$

$$TD = d(R/z)[1-(1-z)^w] \quad [2.60]$$

$$RR = z(R/z)[1-(1-z)^w] \quad [2.61]$$

$$ER = (1-z)(R/z)[1-(1-z)^w] \quad [2.62]$$

$$ID = di(R/z)[1-(1-z)^w] \quad [2.63]$$

$$M\sim = (1+di)(R/z)[1-(1-z)^w] \quad [2.64]$$

The equations [2.58] through [2.64] are for the case of $z \neq 0$. If $z=0$ the above relations will have the following forms;

$$DD + TD = Rw \quad [2.65]$$

$$DD = (1-d)Rw \quad [2.66]$$

$$TD = dRw \quad [2.67]$$

$$RR = 0 \quad [2.68]$$

$$ER = Rw \quad [2.69]$$

$$ID = diRw \quad [2.70]$$

$$M\sim = (1+di)Rw \quad [2.71]$$

For the continuous case (as real number not integer number) of "w" we have the following relations similarly,

$$DD + TD = (R/z)[1-e^{-zw}] \qquad [2.72]$$

$$DD = (1-d)(R/z)[1-e^{-zw}] \qquad [2.73]$$

$$TD = d(R/z)[1-e^{-zw}] \qquad [2.74]$$

$$RR = z(R/z)[1-e^{-zw}] \qquad [2.75]$$

$$ER = (1-z)(R/z)[1-e^{-zw}] \qquad [2.76]$$

$$ID = di(R/z)[1-e^{-zw}] \qquad [2.77]$$

$$M\sim = (1+di)(R/z)[1-e^{-zw}] \qquad [2.78]$$

It is possible now to show that equations [2.64], [2.71] and [2.78] all are similar to equation [2.15]. On the other hand, let rewrite [2.64] as follows;

$$M\sim = (R/z)[1-(1-z)^{w}] +id(R/z)[1-(1-z)^{w}] \qquad [2.79]$$

By replacing [2.58] and [2.63] in [2.79], we will have the following equation;

$$M\sim = DD + (1+id)TD \qquad [2.80]$$

This equation may also be found by replacing "d" from [2.50] into [2.79] and using [2.58] through [2.60].

As it is observed from comparison of [2.80] with [2.15]; both equations are similar in the case that we consider demand deposit (DD) as transaction demand for money (M_T) and time deposit (TD) as speculative demand for money (M_S). However, according to definitions of transaction and speculative motives, demand and time deposits coincide with formers.

2.7.2 Different Reserve Requirement Ratios

In previous sections, we assumed that reserve requirement ratios for demand and time deposits are unique. Now suppose that we have different reserve requirement ratios for different deposits. Let us denote reserve requirement ratios for demand and time deposits by z_D and z_T respectively. By a similar exposition, we had for the case of unique reserve requirement ratio "z" we can develop our discussion for the case that instead of "z" we are confronted with "z_D" and "z_T". Now concentrate on table 2.5 and let z_{DT} denote the following expression, which incorporates the effects of different reserve requirement ratios in the portions of different deposits;

$$z_{DT} = z_D(1-d) + z_T d \qquad [2.81]$$

That is "z_{DT}" is convex combination of "z_D" and "z_T" with factor "d", which is the portion of time deposit to total time and demand deposits as given by [2.50]. For ease of explanation, we may also use "$1-z_{DT}$" by manipulating [2.81] as;

$$1-z_{DT} = 1 - z_D(1-d) - z_T d = (1-z_D)(1-d) + (1-z_T)d \qquad [2.82]$$

Table 2.5

	Demand deposits + time deposits	Demand deposits	Time deposits	Reserve requirement
w	DD+TD	DD	TD	RR
1	R	(1-d)R	dR	$z_{DT}R$
2	$(1-z_{DT})R$	$(1-d)(1-z_{DT})R$	$d(1-z_{DT})R$	$z(1-z_{DT})R$
3	$(1-z_{DT})^2R$	$(1-d)(1-z_{DT})^2R$	$d(1-z_{DT})^2R$	$z(1-z_{DT})^2R$
4	$(1-z_{DT})^3R$	$(1-d)(1-z_{DT})^3R$	$d(1-z_{DT})^3R$	$z(1-z_{DT})^3R$
. . .	. . .	. . .	. . .	. . .
w	$(1-z_{DT})^{w-1}R$	$(1-d)(1-z_{DT})^{w-1}R$	$d(1-z_{DT})^{w-1}R$	$z(1-z_{DT})^{w-1}R$

Table 2.5 continued

	Loan and investment excess reserves	Interest deposits	Total money supply (demand deposit + time deposit + interest deposits)
w	ER	ID=iTD	DD+TD+ID
1	$(1-z_{DT})R$	dRi	R(1+di)
2	$(1-z_{DT})^2R$	$d(1-z_{DT})Ri$	(1-zDT)R(1+di)
3	$(1-z_{DT})^3R$	$d(1-z_{DT})^2Ri$	$(1-zDT)^2R(1+di)$
4	$(1-z_{DT})^4R$	$d(1-z_{DT})^3Ri$	$(1-zDT)^3R(1+di)$
. . .	. . .	. . .	. . .
w	$(1-z_{DT})^wR$	$d(1-z_{DT})^{w-1}Ri$	$(1-z_{DT})^{w-1}R(1+di)$

Explanation of table 2.5 is the same as previous table 2.4 for the case of unique "z". Similar to the series [2.51] through [2.57] we have following series for the case of two different "z_D" and "z_T":

$$DD + TD = R[1+(1-z_{DT})+(1-z_{DT})^2+...+(1-z_{DT})^{w-1}] \quad [2.83]$$

$$DD = (1-d)R[1+(1-z_{DT})+(1-z_{DT})^2+...+(1-z_{DT})^{w-1}] \quad [2.84]$$

$$TD = dR[1+(1-z_{DT})+(1-z_{DT})^2+...+(1-z_{DT})^{w-1}] \quad [2.85]$$

$$RR = zR[1+(1-z_{DT})+(1-z_{DT})^2+...+(1-z_{DT})^{w-1}] \quad [2.86]$$

$$ER = (1-z)R[1+(1-z_{DT})+(1-z_{DT})^2+...+(1-z_{DT})^{w-1}] \quad [2.87]$$

$$ID = diR[1+(1-z_{DT})+(1-z_{DT})^2+...+(1-z_{DT})^{w-1}] \quad [2.88]$$

$$M\sim = (1+di)R[1+(1-z_{DT})+(1-z_{DT})^2+...+(1-z_{DT})^{w-1}] \quad [2.89]$$

If both "z_D" and "z_T" are non-zeros, we may sum above series as following formulas;

$$DD + TD = (R/z_{DT})[1-(1-z_{DT})^w] \quad [2.90]$$

$$DD = (1-d)(R/z_{DT})[1-(1-z_{DT})^w] \quad [2.91]$$

$$TD = d(R/z_{DT})[1-(1-z_{DT})^w] \quad [2.92]$$

$$RR = z(R/z_{DT})[1-(1-z_{DT})^w] \quad [2.93]$$

$$ER = (1-z)(R/z_{DT})[1-(1-z_{DT})^w] \quad [2.94]$$

$$ID = di(R/z_{DT})[1-(1-z_{DT})^w] \quad [2.95]$$

$$M\sim = (1+di)(R/z_{DT})[1-(1-z_{DT})^w] \quad [2.96]$$

If both "z_D" and "z_T" are equal to zero, we again are confronted with [2.65] through

[2.66]. For the case of continuous definition of "w" we have the following formulas instead;

$$DD + TD = (R/z_{DT})[1-e^{(-wz_{DT})}] \quad [2.97]$$

$$DD = (1-d)(R/z_{DT})[1-e^{(-wz_{DT})}] \quad [2.98]$$

$$TD = d(R/z_{DT})[1-e^{(-wz_{DT})}] \quad [2.99]$$

$$RR = z(R/z_{DT})[1-e^{(-wz_{DT})}] \quad [2.100]$$

$$ER = (1-z)(R/z_{DT})[1-e^{(-wz_{DT})}] \quad [2.101]$$

$$ID = di(R/z_{DT})[1-e^{(-wz_{DT})}] \quad [2.102]$$

$$M\sim = (1+di)(R/z_{DT})[1-e^{(-wz_{DT})}] \quad [2.103]$$

Generalization to multi reserve requirements ratios is straightforward and we will not discuss about it anymore.

2.7.3 Times and Velocity of Circulation

In previous section of monetary expansion mechanism, we talked about each round that excess reserve becomes deposit. That is the times of transference of money to bank and then to public. In this regard, we talked about the times of circulation "w". Amount of money transfer in each time is not the same for all times of circulation. Because in each round, some amount of reserves go as reserve requirements. Now we are going to calculate the average of these times of circulation, which according to the definition of velocity of circulation of money "V" means the number of times an "average" unit of money changes hands. To compute velocity we need to compute average of the number of times of money circulation regarding the amount of transfer of money in each time of circulation. According to table 2.3 if "R" units of initial reserve circulate one time, both "w" and "V" are equal to one. That is times and velocity of circulation are equal to one. That is all amounts of "R" circulate fully. In second round when "w" is equal to two; "V" is equal to "1+(1-z)" for one time of circulation of all amount of "R" plus second time of circulation of "(1-z)" amount of "R". This means that velocity is equal to total circulation of reserve "R[1+(1-z)] divided by initial reserve "R". On the other hand, velocity will be equal to "1+(1-z)" as is obvious from table 2.6. Finally for "w" times of circulation of money, total circulated money will be equal to $R[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}]$ as given by [2.28]. Normalizing for one unit of initial reserve "R" comes from dividing total circulated money by "R" which gives the velocity of circulation of money as;

$$V = 1+(1-z)+(1-z)^2+...+(1-z)^{w-1} \quad z\neq 0 \qquad [2.104]$$

That means number of times an average unit of money circulates. Sum of this finite series will be equal to;

$$V = [1-(1-z)^w]/z \quad z\neq 0 \qquad [2.105]$$

When z=0, sum of the series [2.104] will be equal to:

$$V = w \qquad [2.106]$$

Table 2.6

Times of circulation	Total amount of circulated money	Velocity of circulation (Number of times an average unit of money circulates)
w	created deposits	V
1	R	R/R
2	R[1+(1-z)]	R[1+(1-z)]/R
3	$R[1+(1-z)+(1-z)^2$	$R[1+(1-z)+(1-z)^2]/R$
. . .	. . .	. . .
w	$R[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}]$	$R[1+(1-z)+(1-z)^2+...+(1-z)^{w-1}]/R$

If we treat "w" as a real variable as discussed before, velocity of circulation will be equal to the following expression, which simply comes from [2.45] divided by "R";

$$V = (1-e^{-zw})/z \qquad [2.107]$$

Choosing "velocity of circulation" "V" instead of "times of circulation" "w" helps us to connect our discussions to quantity theory of money. Since in the latter, we are confronted with velocity of circulation rather than times of circulation. By replacing [2.105] into our previous derivation of money supply [2.32] through [2.35] gives the following expressions in terms of velocity of circulation of money;

$$DD = RV \qquad [2.108]$$
$$RR = zRV \qquad [2.109]$$
$$ER = (1-z)RV \qquad [2.110]$$
$$RR + ER = RV \qquad [2.111]$$

Since, when z=0, we have V=w as [2.106]. Equations [2.108] through [2.111] will also hold for z=0 which is obvious by considering [2.36] through [2.39]. Applying [2.107] for continuous case of "w" will also give the same results as [2.108] through [2.111]. For the case of existence of demand, time and interest deposits with unique reserve requirement ratios we will find similar results as follows based on [2.58] through [2.64];

$$DD + TD = RV \qquad [2.112]$$
$$DD = (1-d)RV \qquad [2.113]$$
$$TD = d\,RV \qquad [2.114]$$
$$RR = zRV \qquad [2.115]$$
$$ER = (1-z)RV \qquad [2.116]$$
$$ID = diRV \qquad [2.117]$$
$$M\sim = (1+di)RV \qquad [2.118]$$

In above equations "z" can be also equal to zero as cited before. When we have multi reserve requirements ratios the only difference will be in the definition of "z_{DT}" given by [2.81] instead of "z". That is only we replace "z_{DT}" in [2.115] and [2.116] instead of "z".

2.8 Actual and Potential Money Supply

As it was indicated on previous sections, money (deposit) expansion is heavily based on the times of circulation of money. In definition of the times of circulation of money, we always measure the times money changes hand during a period (e.g. year). But the point which is important is that at the next period (year) reserve again circulates and creates money. Suppose central bank has increased total reserves equal to a specific amount at the t^{th} period. This new reserve circulates "w" times and creates money as equations [2.32] or [2.45] states. At the $(t+1)^{th}$ period, total excess reserves as total reserves minus reserve requirement will not be equal to zero. This excess reserve will be zero if reserve has been circulated infinite times. When excess reserve is opposite to zero it has ability to create money (deposit) by circulating through the banking system. In this regard if we had an increment in reserves at previous periods, we are still confronting with its deposit expansionary power at future times. That is actual power of money supply of initial reserve is less than its potential power. Its potential power can be calculated when we allow the initial reserve to circulate infinite times. That is what we stated by [2.25] is “potential power” of money creation of initial reserve, not its “actual power”. The actual money supply comes from the equations [2.32] or [2.45].

This point is very important in performance of tight monetary policy. Suppose that central bank does not change total reserve to preserve money supply unchanged. If the goal of this policy is to control the supply of money, it will not be successful, because it does not change the potential power of deposit creation of previous reserves injected to the economy and the “actual money supply” grows continuously up to “potential money supply” level.

The gap between potential and actual money supply may be simply derived by subtracting [2.26] from [2.32] or [2.45] for different treatments of integer or real forms of times of circulation of money as;

$$GAP = (R/z)(1-z)^{w} \qquad z \neq 0 \qquad [2.119]$$

$$GAP = (R/z)e^{-zw} \qquad z \neq 0 \qquad [2.120]$$

When reserve requirement ratio is zero (z=0), potential money supply will be infinity as [2.27] states and actual money supply will be the same as [2.36]. As it is obvious from [2.119] and [2.120] the gap between potential and actual money supplies will be zero if "w=∞". In equations [2.119] and [2.120] the variable "w" stands for the times that an initial reserve "R" has been circulated since its injection into the economy. In this regard if "R" was injected at time "1" and $w_1, w_2, ..., w_k$ are times of circulation of money at each subsequent periods, amount of "w" in [2.32] through [2.40] and [2.45] through [2.120] will be equal to;

$$w = w_1 + w_2 + ... + w_k \qquad [2.121]$$

Chapter Three

External Monetary Equilibrium

3.1 Foreign Money

Up to this juncture, we talked all about the domestic money as a whole. Now in this section we intend to generalize the model to contain external sector's issues from a monetary point of view. Similar to the international finance theories, monetarism view in this dimension has deep (but not sufficient) roots in the literature, and "The Monetary Approach to Balance of Payments" is the main milestone of the monetarists to express the characteristics of the linkages between domestic and foreign moneys via monetary base. In this section, we are going to restate and reformulate the linkages that exist between domestic and foreign economies in the frame of equilibrium condition of the money and commodity markets. Let's go back to the very important equilibrium equation [2.1]. This equation implies all the essential conditions about the money and commodity markets equilibrium. To analyze the effects of domestic and foreign moneys in the equilibrium condition, we should decompose this equation to some necessary factors. National Income can be decomposed into two main parts of absorption and balance of trade, which absorption is the sum of domestic private and public consumptions and investments. That is if we denote, A : Real absorption

B : Balance of trade in real term

C : Real private consumption

G : Real public consumption

I : Real investment

E_x: Real exports

I_m: Real imports

we can write;

$$y = C + I + G + E_x - I_m = A + B$$

$$A = C + I + G \quad [3.1]$$

$$B = E_x - I_m$$

All the above variables are in domestic money values. On the other side, corresponding to the value of the balance of trade of the economy we should have some demand and supply for foreign money to form the necessary transactions (or speculations). If "M" is the total supply of domestic money, we should have some receipts and payments of foreign money over "M" due to the exports and imports of the home country. It is the foreign money, which is used to facilitate the foreign transactions (or speculations). Within this grouping, domestic money is allocated to absorption and foreign money to foreign trade sector. In this regard, equal to the net balance of trade, we are confronting with some negative or positive (due to deficit or surplus of trade balance) amount of foreign money (net) supply in the economy. Let net volume of foreign currency used to facilitate foreign trade of the local economy in local currency value is denoted by "N", and supply of and demand for foreign currency in local currency units by "S" and "D", then we can write;

$$N = S - D \quad [3.2]$$

where "N", "S" and "D" are all expressed in the unit of domestic currency. When N=0, it means that net supply of foreign money in the local economy is zero. On the other hand, it means that foreign money supply and demand in the local economy are equal. Using [3.1] and [3.2], we can rewrite the equation [2.1] of the quantity theory as the following identity;

$$V(M+N) = P(A+B) \qquad [3.3]$$

Suppose that the price in rest of the world is constant and we have only one foreign currency that can be converted by the exchange rate "e" to domestic currency. The variable "e" is the value of the domestic money expressed as one unit of foreign currency. Supply of, demand for and net demand for foreign currency are simply shown (by lower case letters) in following relations,

$$\begin{aligned} N &= en \\ S &= es \qquad [3.4] \\ D &= ed \end{aligned}$$

Where n, s and d are in units of foreign currency. Similarly for E_x, I_m and B we have;

$$\begin{aligned} B &= eb \\ E_x &= ex \qquad [3.5] \\ I_m &= em \end{aligned}$$

To prevail the equilibrium in the foreign money market we should have;

$$N = B = S - D = E_x - I_m \qquad [3.6]$$

where all variables are in domestic money values. To convert [3.6] to foreign currency by using [3.4] and [3.5] we have;

$$n = b = s - d = x - m \qquad [3.7]$$

Now, we are ready to put the foreign money market into the global equilibrium equation [3.3] by using [3.4] and [3.5] as;

$$V(M+en) = P(A+eb) \qquad [3.8]$$

This identity reveals many facts about the equilibrium condition in the money and commodity markets in domestic and foreign sectors. An implicit assumption is included in [3.8], is the uniqueness of foreign and domestic money velocities – though, intuitively, they should be different. In this regard, one may rewrite [3.8] as the following identity that separates these two velocities;

$$V_d M + V_f en = P(A+eb) \qquad [3.9]$$

where, V_d and V_f denote the velocities of circulation of domestic and foreign money respectively. However, for the sake of simplicity we concentrate on [3.8].

In equation [2.1] of the quantity theory we can simply derive the following relation by dividing the total differential of [2.1] by the equation itself;

$$\hat{}V + \hat{}M = \hat{}P + \hat{}y \qquad [3.10]$$

where the operator "^" denotes the proportionate change. Relation [3.10] expresses that sum of the growth rates of money and velocity is equal to the sum of the growth rates of price and income. By a similar computation on [3.8] we can derive;

$$\hat{}V + \hat{}(M+en) = \hat{}P + \hat{}(A+eb) \qquad [3.11]$$

This again means that sum of the growth rates of velocity and money (domestic and foreign) is equal to the growth rates sum of prices and income (in domestic and foreign sectors). Suppose that velocity, price and income (absorption and trade balance) are constants; therefore, we have,

$$\hat{}(M+en) = \frac{dM + n de + e dn}{M + en} = 0 \qquad [3.12]$$

where (italic) "*d*" indicates differentiation. When domestic money is fixed (*d*M=0) from [3.12] we have,

$$n de + e dn = 0 \implies \hat{}e + \hat{}n = 0 \qquad [3.13]$$

This means that (*Ceteris Paribus*) the growth rate of exchange rate is equal to the negative of the growth rate of the of the foreign money in the home country.

Value of ^n in terms of supply and demand for foreign money can be derived from [3.7]; that is,

$$\hat{}n = \frac{ds - dd}{s - d} = \hat{}(s - d) \qquad [3.14]$$

Substituting [3.14] in [3.13], we will have;

$$d(es) = d(ed) \qquad [3.15]$$

That is total changes in values of supply and demand for foreign money are equal. The Walras' law of market equilibrium condition [3.15] does hold in our money market. Moreover, if we add the assumption of constancy of the total income in the commodity market (right hand side of equation [3.11]) we will have;

$$\hat{}(A+eb) = \frac{dA + b de + e db}{A + eb} = 0 \qquad [3.16]$$

When absorption is also constant [3.16], reduces to;

$$b de + e db = 0 \implies \hat{}e + \hat{}b = 0 \qquad [3.17]$$

^b in terms of export and import of commodity can be derived from [3.7] as;

$$\hat{b} = \frac{dx - dm}{x - m} = \hat{}(x - m) \qquad [3.18]$$

Substituting [3.18] in [3.17], we will have;

$$d(ex) = d(em) \qquad [3.19]$$

which is again the equilibrium condition in the foreign trade commodity market.

Multimarket equilibrium in foreign money and foreign trade markets are derived by equating [3.17] and [3.13] that expresses,

$$\hat{b} = \hat{n} \qquad [3.20]$$

Simply, we could derive this condition from [3.7], but our main goal to choose another way was to express equilibrium in different markets separately.

3.2 Exchange Rate Determinants

Assume that foreign world is in a constant situation. Prices, income, quantity of money, interest rate in the foreign economy are all constants. In this case, we are going to check the determinants of exchange rate, which are influenced by the domestic economic variables. Total differentiation of [3.8] is,

$$(M+en)dV+VdM+Vnde+Vedn = (A+eb)dP+PdA+Pbde+Pedb \qquad [3.21]$$

Rearrange the terms,

$$(Vn-Pb)de = e[d(Pb)-d(Vn)] + [d(PA)-d(VM)] \qquad [3.22]$$

From equation [3.8] we may write,

$$PA - VM = e(Vn - Pb) \qquad [3.23]$$

Divide both sides of [3.22] by e(Vn-Pb)

$$\frac{de}{e} = -\frac{d(\mathrm{Pb})-d(\mathrm{Vn})}{\mathrm{Pb} - \mathrm{Vn}} + \frac{d(\mathrm{PA})-d(\mathrm{VM})}{e(\mathrm{Vn}-\mathrm{Pb})} \qquad [3.24]$$

Using [3.23], the denominator of the last expression of [3.24] can be changed to PA-VM. Therefore, we can write,

$$\hat{e} = \hat{(\mathrm{VM}-\mathrm{PA})} - \hat{(\mathrm{Vn}-\mathrm{Pb})} \qquad [3.25]$$

Equation [3.25] expresses that rate of change of exchange rate is equal to difference of two rates of changes in domestic and foreign sectors imbalances in money and commodity markets. Note that VM-PA is the imbalance of domestic commodity (absorption) and domestic money markets. And also Vn-Pb is the corresponding imbalance in the foreign commodity and foreign money markets (in the local economy). Rate of change of exchange rate ^e is difference of the rates of changes of these two imbalances. In order to fix the exchange rate (^e=0), necessarily we should have ^(Vn-Pb)=^(VM-PA), that means the rate of change of these two imbalances are equal that also makes sense intuitively.

On the other side, we can deduce the exchange rate from equation [3.23] as;

$$e = \frac{\mathrm{PA} - \mathrm{VM}}{\mathrm{Vn} - \mathrm{Pb}} \qquad [3.26]$$

The numerator and denominator of [3.26] just consider the internal and external sectors of the economy respectively. In this regard, [3.26] expresses that exchange rate is equal to the ratio of internal and external imbalances in economy. Internal imbalance comes from disconformity of absorption value (PA) and money value (VM). External imbalance comes from disconformity of net values of foreign money circulating in local economy and balance of trade (both in foreign currency units).

3.3 Price Determinants

Again, assume that the assumptions of previous section hold. Rearranging the terms in [3.21], we have;

$$(A+eb)dP = - Pd(A+eb) + (M+en)dV + Vd(M+en) \qquad [3.27]$$

Divide both sides of [3.27] by P(A+eb) to find,

$$\frac{dP}{P} = - \frac{d(A+eb)}{A+eb} + \frac{(M+en)dV}{P(A+eb)} + \frac{Vd(M+en)}{P(A+eb)} \qquad [3.28]$$

Using [3.8], rate of change of price will be equal to:

$$\hat{}P = \hat{}V + \hat{}(M+en) - \hat{}(A+eb) \qquad [3.29]$$

That is, rate of change of price is equal to rate of change of velocity plus rate of change of domestic and foreign monies minus rate of change of income (including domestic and foreign).

3.4 Different Demand Motives for Foreign Currency

Similar to the domestic currency there are precautionary, transaction and speculative motives for foreign currency. In this regard, one may develop the equilibrium condition [3.8] by using this notion that the foreign money in domestic economy acts like local currency. The precautionary, transaction and speculative demands all occur for the foreign money as well as domestic money. This is not an unfamiliar proposition and due to the strength and reliability of some of foreign currencies, sometimes, these foreign currencies have better acceptability than local currency. In the same manner, existence of various banking facilities for foreign currency make the foreign currencies as interest bearing asset.

Therefore, we can consider the domestic and foreign money markets simultaneously. Without any elaboration, we may again define the three demand motives for foreign currency as well as domestic one. Hence, equation [2.15] will hold for local money and we should write a similar equation for foreign money as follows;

$$n = n_T + (1+i)n_S \qquad [3.30]$$

where "n" and "i" defined earlier, n_T and n_S are the amounts of foreign currency that are used for transaction and speculation purposes. Equilibrium equation of [3.8] can be modified by using [2.15] and [3.30] as;

$$V\{M_T + (1+i)M_S + e[n_T + (1+i)n_S]\} = P(A+eb) \qquad [3.31]$$

This relation also can be written in a manner that explains the precautionary and non-precautionary portions of demand for foreign and domestic money as;

$$(1-r)V\{M_T+(1+i)M_S+e[n_T+(1+i)n_S]\}+rV\{M_T+(1+i)M_S+e[n_T+(1+i)n_S]\}=P(A+eb) \qquad [3.32]$$

Equations [3.31] or [3.32] show all the real and monetary variables in the equilibrium condition. The expression $M_T+(1+i)M_S$ and $e[n_T+(1+i)n_S]$ are supply amounts of local and foreign moneys respectively. By rewriting equation [3.31] as;

$$V[(M_T+en_T) + (1+i)(M_S+en_S)] = P(A+eb) \qquad [3.33]$$

We also can observe the distribution of transaction and speculation demands for home and foreign moneys as we had in our simpler model [2.14]. The equation [3.33] is again the equilibrium condition of money and commodity markets when we have foreign trade and foreign currency in the economy accompanying with domestic money and commodity markets and an interest rate. It also shows that how external sector of the economy interferes in the economy and how the economy reaches equilibrium.

Chapter Four

International Monetary Equilibrium

4.1 Equilibrium Condition

In this part, we are considering the international monetary equilibrium condition. Suppose there are only two countries in the world denoted by home and foreign (rest of the world) as well. Reserve the superscripts H and F for these two countries respectively. Let us write the definitions of our new variables as follows;

V^H, V^F: Velocity of circulation of money in home and foreign countries.

P^H, P^F: Price index of income in home and foreign countries.

A^H, A^F: Volume of absorption in home and foreign countries.

B^H, B^F: Volume of trade balance in home and foreign countries.

e : Home/foreign currencies exchange rate.

M^H: Total supply of home money (in home currency units).

M^F: Total supply of foreign money (in foreign currency units).

M^{HF}: Volume of home money circulating in foreign country (in home currency units).

M^{FH}: Volume of foreign money circulating in home country (in foreign currency units).

M^{HH}: Volume of home money circulating in home country (in home currency units).

M^{FF}: Volume of foreign money circulating in foreign country (in foreign currency units).

Let us deal with this important phenomenon that some portions of national money of one country circulate in the other countries. In this regard, this portion of money reduces the supply of money in home country and inversely increases the supply of money in foreign country, because foreign money circulates in home country as home currency does. Thus, supply of money in home country is equal to the net remained domestic money supply plus a portion of foreign money, which has different nominal value. For the foreign country, there is a similar discussion too. Therefore, we may write the money in circulation in the home and foreign countries as:

$$M^{CH} = M^{HH} + eM^{FH} = M^{H} - M^{HF} + eM^{FH} \qquad [4.1]$$

$$M^{CF} = M^{FF} + M^{HF}/e = M^{F} - M^{FH} + M^{HF}/e \qquad [4.2]$$

M^{CH} in [4.1] is sum of the home and foreign moneys circulating in home country evaluated in terms of home money unit and M^{CF} in [4.2] is sum of the foreign and home moneys circulating in foreign country evaluated in terms of foreign money. It would not be a strong assumption to consider that volume of home money circulating in foreign country be equal to the volume of foreign money circulating in home country multiplied by the exchange rate. That is, sum of the two last terms of the right hand side of [4.1] should be zero. Similarly, in [4.2] this should be occurred inversely, that is, the volume of foreign money circulating in home country should be equal to the home money circulating in foreign country divided by the exchange rate. This means again that sum of the two last terms of the right hand side of [4.2] should be zero. However, this proposition makes sense intuitively, moreover in the next section; we will prove where it will be true and will introduce an interesting monetary rule to determine the exchange rate. Now, at this section we continue without using this condition. Let us write the equilibrium condition in money and commodity markets in both countries. Similar to [3.3] for home and foreign countries we may write,

$$V^H M^{CH} = P^H(A^H+B^H) \qquad [4.3]$$

$$V^F M^{CF} = P^F(A^F+B^F) \qquad [4.4]$$

which have this presumption implicitly that velocities of circulation of home and foreign moneys in both home and foreign countries are equal. This does not mean that velocity of money in home is the same as foreign. Deletion of this assumption does not distort the frame of reasoning and one may rebuild the models when this assumption is deleted. Using [4.1] and [4.2] we may write the above relations as follows;

$$V^H(M^H - M^{HF} + eM^{FH}) = P^H(A^H + B^H) \qquad [4.5]$$

$$V^F(M^F - M^{FH} + M^{HF}/e) = P^F(A^F + B^F) \qquad [4.6]$$

Let us look for the equilibrium condition of money and commodity in international market. The left hand sides of equations [4.5] and [4.6] present the money sides of domestic and foreign economies and the right hand sides present commodity sides. Total value of money and total value of income in the world can be derived by summing these two equations. For the sake of simple derivations, before summation multiply both sides of [4.6] by "e". Therefore;

$$V^H(M^H-M^{HF}+eM^{FH})+V^F(eM^F-eM^{FH}+M^{HF}) = P^H(A^H+B^H)+eP^F(A^F+B^F) \qquad [4.7]$$

The international equilibrium condition will be;

$$V^H M^H+eV^F M^F+(V^F-V^H)(M^{HF}-eM^{FH}) = P^H(A^H+B^H)+eP^F(A^F+B^F) \qquad [4.8]$$

This means total value of the world money will be equal to values of money circulating in home and foreign countries ($V^H M^H + eV^F M^F$) plus net value of transferred money between two countries ($(V^F - V^H)(M^{HF} - eM^{FH})$). Total value of income is equal to sum of incomes in both countries (the right hand side of [4.8]).

4.1.1 Exchange Rate Determinants

One can simply derive the exchange rate variable from [4.8]. This will be equal to;

$$e = \frac{V^H(M^{HF} - M^H) - V^F M^{HF} + P^H(A^H + B^H)}{V^F(M^F - M^{FH}) + V^H M^{FH} - P^F(A^F + B^F)} \qquad [4.9]$$

Since,

$$M^{HH} = M^H - M^{HF}; \quad M^{FF} = M^F - M^{FH} \qquad [4.10]$$

we may write [4.9] as,

$$e = \frac{- V^H M^{HH} - V^F M^{HF} + P^H(A^H + B^H)}{V^F M^{FF} + V^H M^{FH} - P^F(A^F + B^F)} \qquad [4.11]$$

On the other side, balance of trade in home country should be equal to the negative of balance of trade in foreign country multiplied by exchange rate, that is;

$$B^H = -eB^F \equiv B \qquad [4.12]$$

Which we denoted "B" as balance of trade in home currency units. Substituting [4.12] in [4.8] and solving for "e" gives,

$$e = \frac{V^H(M^{HF} - M^H) - V^F M^{HF} + P^H A^H + (P^H - P^F)B}{V^F(M^F - M^{FH}) + V^H M^{FH} - P^F A^F} \qquad [4.13]$$

Then by using [4.10] in [4.13] we have;

$$e = \frac{- V^H M^{HH} - V^F M^{HF} + P^H A^H + (P^H - P^F)B}{V^F M^{FF} + V^H M^{FH} - P^F A^F} \qquad [4.14]$$

This equation determines the exchange rate at international level. Now assume that velocities of circulation of moneys in both countries are equal to V, that is;

$$V^H = V^F \equiv V \qquad [4.15]$$

Substituting [4.15] in [4.14] and by using [4.8] the exchange rate will be equal to:

$$e = \frac{- VM^H + P^H A^H + (P^H - P^F)B}{VM^F - P^F A^F} \qquad [4.16]$$

If trade is on balance and B=0, then the exchange rate will be equal to:

$$e = \frac{P^H A^H - VM^H}{VM^F - P^F A^F} \qquad [4.17]$$

Now, assume that trade prevails and classical trade theories assumptions (such as zero transportation costs) are valid. Therefore, we may accept uniqueness of prices at international level as;

$$P^H = P^F \equiv P \qquad [4.18]$$

In this case, equation [4.16] will be changed to:

$$e = \frac{PA^H - VM^H}{VM^F - PA^F} \qquad [4.19]$$

This relation to some extent is similar to [3.26] with some additional justifications. Numerator of the right hand sides of both relations are disconformities in absorption value and domestic money value in the home country. The denominators are related to foreign sector. In [3.26] the Vn-Pb is the effect of disconformity in foreign money and trade - in the case that the constructing assumptions of [3.26] (the interrelationship of home and foreign countries are in their net balance of trade and the corresponding net money payments) are prevailed. But, in [4.19], the denominator shows the effect of foreign money and income in determining the exchange rate in a more general case when trade is balanced.

Now, totally differentiate [4.19] and divide the result by [4.19] itself, we will have,

$$\hat{e} = \hat{}(VM^{H}-PA^{H}) - \hat{}(VM^{F}-PA^{F}) \qquad [4.20]$$

Which means the rate of change of exchange rate is equal to the difference of two disconformities in flows of money and commodity in both home and foreign countries. To fix the exchange rate at international level we should have $\hat{}(VM^{H}-PA^{H})=\hat{}(VM^{F}-PA^{F})$. But, this is the case when we postulated the assumptions of [4.15], [4.18] and B=0. In the more general case of [4.14] we can derive the rate of change of exchange rate as;

$$\hat{e}=\hat{}[V^{H}M^{HH}+V^{F}M^{HF}-P^{H}A^{H}-(P^{H}-P^{F})B]-\hat{}[V^{F}M^{FF}+V^{H}M^{FH}-P^{F}A^{F}] \qquad [4.21]$$

4.1.2 Simple Exchange Rate Monetary Rule

In the earlier sections after presentation of equations [4.1] and [4.2], we pointed out that the two last terms of the right hand sides of these equations may be canceled out at equilibrium. We stated that one might say that volume of home money circulating in foreign country must be equal to the volume of foreign money circulating in home country multiplied by the exchange rate and conversely, the volume of foreign money circulating in home country should be equal to the home money circulating in foreign country divided by the exchange rate. However, at that section we left the proposition that makes sense intuitively. In this section, we are going to prove that this proposition holds if the velocities of circulation of home and foreign moneys be equal.

If this proposition were true, we had following two equations from our discussions on [4.1] and [4.2];

$$- M^{HF} + eM^{FH} = 0 \qquad [4.22]$$

$$- M^{FH} + M^{HF}/e = 0 \qquad [4.23]$$

Both of the above equations can be solved for "e". Both equations give the same following solution as;

$$e = \frac{M^{HF}}{M^{FH}} \qquad [4.24]$$

This means that exchange rate is equal to the ratio of the volumes of home money circulating in foreign country to the volume of foreign money circulating in home country. Now, we are going to prove that [4.24] is true by an inverse proof. If [4.24] is true, then [4.22] and [4.23] will be true. So from [4.1] and [4.2] (using [4.24] we have;

$$M^{CH} = M^{H} \qquad [4.25]$$

$$M^{CF} = M^{F} \qquad [4.26]$$

The equations [4.3] and [4.4] both remain valid and instead of [4.5] and [4.6] we have the following equations;

$$V^{H}M^{H} = P^{H}(A^{H}+B^{H}) \qquad [4.27]$$

$$V^{F}M^{F} = P^{F}(A^{F}+B^{F}) \qquad [4.28]$$

Similar to the procedure we had before to derive [4.7], we multiply both sides of [4.28] by "e" and sum the resulted equation with [4.27]. The following is simply derived;

$$V^H M^H + eV^F M^F = P^H(A^H+B^H) + eP^F(A^F+B^F) \qquad [4.29]$$

Corresponding to [4.9] we have the following equation,

$$e = \frac{- V^H M^H + P^H(A^H+B^H)}{V^F M^F - P^F(A^F+B^F)} \qquad [4.30]$$

To compare [4.30] with [4.9], one may rewrite [4.9] by rearranging it simply as;

$$e = \frac{(V^H-V^F)M^{HF} - V^H M^H + P^H(A^H+B^H)}{(V^H-V^F)M^{FH} + V^F M^F - P^F(A^F+B^F)} \qquad [4.31]$$

This equation will be equal to [4.30] if we have one of the following conditions;

$$M^{HF} = M^{FH} = 0 \qquad [4.32]$$

or,

$$(V^H-V^F) = 0 \qquad [4.33]$$

The condition [4.32] is trivial, but [4.33] is non-trivial. If [4.33] holds, that is velocity of circulation of home and foreign money be equal, then [4.30] is equal to [4.31] or [4.9]. This means that, in the case of equal velocities of circulation of money in home and foreign countries, exchange rate is determined by the ratio of the volume of the home money circulating in foreign country to the volume of foreign money circulating in home country, or the ratio given by [4.24].

Now let's continue by entering balance of trade condition of [4.12] to recheck the rule of [4.24]. By using this condition on [4.27] and [4.28] we reach,

$$e = \frac{- V^H M^H + P^H A^H + (P^H-P^F)B}{V^F M^F - P^F A^F} \qquad [4.34]$$

If, again, apply the condition of equal velocities in home and foreign countries; from [4.34], then we will find equation [4.16]. This again reconfirms the necessity of equal velocities condition to establish the ratio of [4.24]. Other relations of [4.17] and [4.19] can be derived straightforwardly. However, in this section, we showed that the simple exchange rate monetary rule given by [4.24] is satisfied if the velocities of circulation of home and foreign moneys be equal. In the next section, we are going to find a more general rule for the case that velocities are different.

4.1.3 General Exchange Rate Monetary Rule

In continuation of the last section, we are going to propose that in our earlier frame of analysis, the exchange rate is determined by the following general exchange rate monetary rule;

$$e = \frac{V^F M^{HF}}{V^H M^{FH}} \qquad [4.35]$$

That is the exchange rate is determined by the ratio of velocity of circulation of foreign money times home money circulating in foreign country ($V^F M^{HF}$) to velocity of circulation of home money times foreign money circulating in home country ($V^H M^{FH}$). According to our previous interpretation of velocity of circulation as price of money, the numerator of [4.35] is value of home money circulating in foreign country (value = price × volume) and the denominator of [4.35] is the value of foreign money circulating in home country. In this regard [4.35] presents that the exchange rate is the "relative values of home money circulating in foreign country to foreign money circulating in home country". This proposition makes our general exchange rate monetary rule complete.

Now, let us prove our proposition given by [4.35]. For this proof, we should show that application of [4.35] on [4.5] and [4.6] gives the same results as we had before in [4.9] through [4.19]. By replacing [4.35] in [4.5] and [4.6] we will have;

$$V^H\left(M^H - M^{HF} + \frac{V^F}{V^H} M^{HF}\right) = P^H(A^H + B^H) \qquad [4.36]$$

$$V^F\left(M^F - M^{FH} + \frac{V^H}{V^F} M^{FH}\right) = P^F(A^F + B^F) \qquad [4.37]$$

After multiplication we have;

$$V^H M^H - V^H M^{HF} + V^F M^{HF} = P^H(A^H + B^H) \qquad [4.38]$$

$$V^F M^F - V^F M^{FH} + V^H M^{FH} = P^F(A^F + B^F) \qquad [4.39]$$

Multiply both sides of [4.39] by "e" and sum the result with [4.38]. The result will be;

$$V^H M^H - V^H M^{HF} + V^F M^{HF} + e(V^F M^F - V^F M^{FH} + V^H M^{FH}) = P^H(A^H + B^H) + eP^F(A^F + B^F) \qquad [4.40]$$

Solving [4.40] for "e" gives;

$$e = \frac{- V^H M^H + V^H M^{HF} - V^F M^{HF} + P^H(A^H + B^H)}{V^F M^F - V^F M^{FH} + V^H M^{FH} - P^F(A^F + B^F)} \qquad [4.41]$$

This equation is equal to [4.9] and the proposition was proved. Therefore, other equations as [4.11] through [4.19] all are valid for our general exchange rate monetary rule.

4.2 Interest Rate and International Monetary Equilibrium

In this section, we enter rate of interest into our international monetary equilibrium model. Given our later discussions on transaction and speculative motives, we may develop the relation [4.7] to include the interest rates in home and foreign countries denoted by i^H and i^F respectively. Reserve the subscripts "T" and "S" for transaction and speculative portions of moneys respectively in both home and foreign countries. We may decompose circulating moneys in home and foreign countries in [4.1] and [4.2] as

$$M^{CH} = M^{CH}{}_T + (1+i^H)M^{CH}{}_S \qquad [4.42]$$

$$M^{CF} = M^{CF}{}_T + (1+i^F)M^{CF}{}_S \qquad [4.43]$$

Existence of 1 and i in the parentheses of $(1+i^H)$ and $(1+i^F)$ are due to the amount of money and the interest created at the end of period. We also discussed about this specification at previous chapters. On the other hand,

$$M^{CH} = M^{HH}{}_T + eM^{FH}{}_T + (1+i^H)(M^{HH}{}_S+eM^{FH}{}_S) \qquad [4.44]$$

$$M^{CF} = M^{FF}{}_T + M^{FH}{}_T/e + (1+i^F)(M^{FF}{}_S+M^{HF}{}_S/e) \qquad [4.45]$$

Definitional relations [4.44] and [4.45] can be written as;

$$M^{CH} = M^H{}_T + eM^{FH}{}_T + (1+i^H)(M^H{}_S+eM^{FH}{}_S) - M^{HF} \qquad [4.46]$$

$$M^{CF} = M^F{}_T + M^{HF}{}_T/e + (1+i^F)(M^F{}_S+M^{HF}{}_S/e) - M^{FH} \qquad [4.47]$$

But, according to our previous discussions on different money motives, M^{HF} in [4.46] and M^{FH} in [4.47] should be equal to:

$$M^{HF} = M^{HF}{}_T + (1+i^F)M^{HF}{}_S \qquad [4.48]$$

$$M^{FH} = M^{FH}{}_T + (1+i^H)M^{FH}{}_S \qquad [4.49]$$

Substitution of [4.48] and [4.49] into [4.46] and [4.47] gives;

$$M^{CH} = M^{H}_{T}+eM^{FH}_{T}+(1+i^{H})(M^{H}_{S}+eM^{FH}_{S})-M^{HF}_{T}-(1+i^{F})M^{HF}_{S} \quad [4.50]$$

$$M^{CF} = M^{F}_{T}+M^{HF}_{T}/e+(1+i^{F})(M^{F}_{S}+M^{HF}_{S}/e)-M^{FH}_{T}-(1+i^{H})M^{FH}_{S} \quad [4.51]$$

These equations also show the negative effect of increasing interest rate in money supply of the opposite country.

Using [4.3] and [4.4] and [4.50] and [4.51], we may write the international monetary equilibrium conditions similar to [4.5] and [4.6] as;

$$V^{H}[M^{H}_{T}+eM^{FH}_{T}+(1+i^{H})(M^{H}_{S}+eM^{FH}_{S})-M^{HF}_{T}-(1+i^{F})M^{HF}_{S}] = P^{H}(A^{H}+B^{H}) \quad [4.52]$$

$$V^{F}[M^{F}_{T}+M^{HF}_{T}/e+(1+i^{F})(M^{F}_{S}+M^{HF}_{S}/e)-M^{FH}_{T}-(1+i^{H})M^{FH}_{S}] = P^{F}(A^{F}+B^{F}) \quad [4.53]$$

Multiply [4.53] by "e" and sum the resulting relation by [4.52] leads us to international monetary equilibrium condition similar to [4.7].

$$\begin{aligned} &V^{H}[M^{H}_{T}+eM^{FH}_{T}+(1+i^{H})(M^{H}_{S}+eM^{FH}_{S})-M^{HF}_{T}-(1+i^{F})M^{HF}_{S}] + \\ &V^{F}[eM^{F}_{T}+M^{HF}_{T}+ (1+i^{F})(eM^{F}_{S}+M^{HF}_{S})-eM^{FH}_{T}-(1+i^{H})eM^{FH}_{S}] = \\ &\qquad P^{H}(A^{H}+B^{H}) + eP^{F}(A^{F}+B^{F}) \quad [4.54] \end{aligned}$$

This is the general international monetary equilibrium condition for two countries. Precautionary and non-precautionary portions can be derived simply as before in form of convex decomposition of left side of [4.54] with the risk rate "r".

4.2.1 Exchange Rate Determinants

By solving [4.54] for "e" we have the exchange rate equation as follows:

$$E = \frac{-V^{H}[M^{H}_{T}-M^{HF}_{T}+(1+i^{H})M^{H}_{S}-(1+i^{F})M^{HF}_{S}]-V^{F}[M^{HF}_{T}+(1+i^{F})M^{HF}_{S}] + P^{H}(A^{H}+B^{H})}{V^{F}[M^{F}_{T}-M^{FH}_{T}+(1+i^{F})M^{F}_{S}-(1+i^{H})M^{FH}_{S}]+V^{H}[M^{FH}_{T}+(1+i^{H})M^{FH}_{S}] - P^{F}(A^{F}+B^{F})} \quad [4.55]$$

Now, let us simplify the above equation by using the relation [4.12]. Substituting [4.12] in [4.54] and solving again for "e" we have;

$$e = \frac{-V^H[M^H{}_T-M^{HF}{}_T+(1+i^H)M^H{}_S-(1+i^F)M^{HF}{}_S]-V^F[M^{HF}{}_T+(1+i^F)M^{HF}{}_S] + P^HA^H+(P^H-P^F)B}{V^F[M^F{}_T-M^{FH}{}_T+(1+i^F)M^F{}_S-(1+i^H)M^{FH}{}_S]+V^H[M^{FH}{}_T+(1+i^H)M^{FH}{}_S] - P^FA^F} \quad [4.56]$$

Assume that the velocities of circulation of moneys are equal in home and foreign countries; by using [4.15] the exchange rate equation [4.56] will be reduced to:

$$e = \frac{- V[M^H{}_T+(1+i^H)M^H{}_S] + P^HA^H + (P^H-P^F)B}{V[M^F{}_T+(1+i^F)M^F{}_S] - P^FA^F} \quad [4.57]$$

When trade is on balance, then B=0 and we have;

$$e = \frac{- V[M^H{}_T+(1+i^H)M^H{}_S] + P^HA^H}{V[M^F{}_T+(1+i^F)M^F{}_S] - P^FA^F} \quad [4.58]$$

In the case of unique international price [4.18], from [4.57] we have;

$$e = \frac{- V[M^H{}_T+(1+i^H)M^H{}_S] + PA^H}{V[M^F{}_T+(1+i^F)M^F{}_S] - PA^F} \quad [4.59]$$

4.2.2 Simple Exchange Rate Monetary Rule and Interest Rate

In this section, we are going to develop the simple exchange rate monetary rule to a more general case, which includes interest rate. In the former simple rule in the last section, we were confronted with equation [4.24]. We observed that this equation holds when we work on a frame that different demand motives and corresponding interest rate do not enter the quantity theory equation. Now, in this section, we are going to check the effect of interest rate in our simple exchange rate monetary rule.

By using [4.48] and [4.49] we can write the simple rule equation of [4.24] as follows;

$$e = \frac{M^{HF}}{M^{FH}} = \frac{M^{HF}{}_T + (1+i^F)M^{HF}{}_S}{M^{FH}{}_T + (1+i^H)M^{FH}{}_S} \qquad [4.60]$$

We showed that the left hand side equality of [4.60] holds when velocities of circulation of money in foreign and home countries are equal, and here we should prove that with the same circumstances, the right hand side equality of [4.60] also holds. Therefore, from [4.60] we can write;

$$M^{HF}{}_T + (1+i^F)M^{HF}{}_S - e[M^{FH}{}_T + (1+i^H)M^{FH}{}_S] = 0 \qquad [4.61]$$

$$[M^{HF}{}_T + (1+i^F)M^{HF}{}_S]/e - M^{FH}{}_T - (1+i^H)M^{FH}{}_S = 0 \qquad [4.62]$$

By replacing the two above equations in [4.52] and [4.53] we find,

$$V^H[M^H{}_T + (1+i^H)M^H{}_S] = P^H(A^H + B^H) \qquad [4.63]$$

$$V^F[M^F{}_T + (1+i^F)M^F{}_S] = P^F(A^F + B^F) \qquad [4.64]$$

Multiply both sides of [4.64] by "e" and sum the results with [4.63], gives:

$$V^H[M^H{}_T+(1+i^H)M^H{}_S] + eV^F[M^F{}_T+(1+i^F)M^F{}_S] = P^H(A^H+B^H) + eP^F(A^F+B^F) \qquad [4.65]$$

Solving this equation for "e" leads to;

$$e = \frac{- V^H[M^H{}_T + (1+i^H)M^H{}_S] + P^H(A^H + B^H)}{V^F[M^F{}_T + (1+i^F)M^F{}_S] - P^F(A^F + B^F)} \qquad [4.66]$$

Let us rewrite [4.55] to become more compatible with [4.66] as follows;

$$e = \frac{-V^H[M^H{}_T + (1+i^H)M^H{}_S] + P^H(A^H+B^H) + (V^H-V^F)[M^{HF}{}_T + (1+i^F)M^{HF}{}_S]}{V^F[M^F{}_T + (1+i^F)M^F{}_S] - P^F(A^F+B^F) + (V^H-V^F)[M^{FH}{}_T + (1+i^H)M^{FH}{}_S]} \quad [4.67]$$

In order that [4.66] be equal to [4.55] or [4.67], we should have a unique velocities of circulation of home and foreign moneys (as a non-trivial solution) again as [4.33]. Therefore, if this is the case that [4.60] is satisfied, all of the equations of [4.56] through [4.59] also remain true for our [4.60] and the proposition was proved. Thus, we may again accept [4.60] as our simple exchange rate monetary rule which includes home and foreign interest rates. Simply, this rule states how foreign and home interest rates interfere on exchange rate determination.

4.2.3 General Exchange Rate Monetary Rule and Interest Rate

In this section, we are going to develop the general exchange rate monetary rule when interest rates present in the model and velocities of circulation of money in foreign and home countries are not equal. In this case, similar to [4.35] our general rule (using [4.48] and [4.49]) can be rewritten as follows;

$$e = \frac{V^F[M^{HF}{}_T + (1+i^F)M^{HF}{}_S]}{V^H[M^{FH}{}_T + (1+i^H)M^{FH}{}_S]} \quad [4.68]$$

To verify the accuracy of [4.68] we should choose the same procedure as before. First, replace for "e" in [4.52] and [4.53] to find;

$$V^H[M^H{}_T+(1+i^H)M^H{}_S]+V^F[M^{HF}{}_T+(1+i^F)M^{HF}{}_S]-V^H[M^{HF}{}_T+(1+i^F)M^{HF}{}_S] \\ =P^H(A^H+B^H) \quad [4.69]$$

$$V^F[M^F{}_T+(1+i^F)M^F{}_S]+V^H[M^{FH}{}_T+(1+i^H)M^{FH}{}_S]-V^F[M^{FH}{}_T+(1+i^H)M^{FH}{}_S] \\ =P^F(A^F+B^F) \quad [4.70]$$

Multiply both sides of [4.70] by "e" and then sum the resulted equation with [4.69]. Then we will have;

$$V^H[M^H_T+(1+i^H)M^H_S]+V^F[M^{HF}_T+(1+i^F)M^{HF}_S]-V^H[M^{HF}_T+(1+i^F)M^{HF}_S] + e\{V^F[M^F_T+(1+i^F)M^F_S]+V^H[M^{FH}_T+(1+i^H)M^{FH}_S]-V^F[M^{FH}_T+(1+i^H)M^{FH}_S]\} = P^H(A^H+B^H)+eP^F(A^F+B^F) \qquad [4.71]$$

The solution of [4.71] for "e" will be the same as [4.55] exactly. Therefore, equations [4.56] through [4.59] can be derived sequentially for the case of [4.68] condition and the proof is complete. Therefore, we can observe that general exchange rate monetary rule of [4.68] is valid. Obviously, we can see from [4.68] that foreign interest rate changes will change exchange rate in the same direction but home interest rate changes will change the exchange rate in opposite direction.

One can extend all of the earlier discussions in this section to a multi countries model.

Chapter Five

Transaction and Income

Up to this juncture, we had assumed that real income is a suitable scale variable for total volume of transactions. This assumption forced us to choose the revised form of Fisher's quantity theory in our previous analysis. In this regard, we used the equation [2.1] that its right hand side was "Py" instead of "Pt", where "t" stands for total volume of transactions in the economy. In this section, now, we are going to determine the exact relationship between these two fundamental variables. On the other hand, it is tried to bridge between Fisher's original quantity theory (MV=Pt) and revisionists' interpretation of quantity theory (MV=Py) in a logical frame.

5.1 Value of Transactions in Production Process

To find out the relationship of value added to the amount of nominal payment required performing corresponding transactions; we try to follow the procedure that value added is produced in the economy. Before going through discussions, it should be cited that in all of procedures of national income accounting, we accumulate value added produced by any economic agent of the economy. But, necessarily, production in national income accounting does not mean creating a physical product. In general term, any transaction produces positive or negative value added and the amount of value added is calculated when transaction occurs. This is the case if we adopt each of the three important procedures of national income accounting that are income side, expenditure side and value added side calculations. Another point to remember is this argument that in macro frame of analysis we have only one type of commodity under the title "value added". Therefore, in this regard we just touch value added as the only macro-commodity that is produced in the economy. Before going through the discussion of the relationship between transaction and income, we should assume that trade is in balance and the quantity and value of imports are equal to those of exports. This assumption is due to the transactional nature of imports and exports. That is imports and exports of goods and services are equal to their transaction values, but net exports (exports minus imports) implies value added. However, we will release this assumption later.

However, we try to examine two extreme processes of value added production and then mix them together to reach an adjusted one for operational works.

5.1.1 Integrated Production Process

By this title, we mean that process of production of "value added macro commodity" is sequential and value added is produced by using previously produced value added as input. Suppose that, there exit many firms and they only produce one commodity, namely, "value added". Each firm receives input (in terms of value added) from the previous firm and gives output (in terms of value added) to the next firm. In this case, cost of production of the last firm is equal to accumulation of costs of all previous firms. This means that the last produced value added includes all of the previously produced values added as input or cost. In contrast to this process, we will refer to disintegrated production process in the next section. This latter process considers again many firms, but with the characteristic that the production of one firm necessarily is not used as input for the next firm. However, these two processes will be thoroughly explained in the text. Here, let us go through the first process of integrated value added production.

Suppose that an agent (or firm) (say agent number zero) in the economy possesses a commodity (call it C_0) having price of P_0 in the market. This agent sales his commodity to another agent (number one) with a new price of P_1 that this new price is equal to the previous price (P_0) plus some earned profits (value added) (call it Y_1) (by agent number zero). That is,

$$P_1 = P_0 + Y_1 \quad [5.1]$$

The amount of Y_1 is the value added of this transaction. But, to perform this transaction, agent number one should pay P_1 units of money to agent number zero. Thus, the amount of required money payment to create Y_1 units of value added is equal to P_1 units. If we denote T_1 as value of transaction at the first round, then we will have the following equation;

$$T = T_1 = P_1 \quad [5.2]$$

Where "T" is total value of transactions. In this process, in the national income accounting framework we call that commodity C_1 has been produced and at this moment belongs to the agent number one.

Total value added at this round is equal to Y_1. Denote "Y" as total nominal value added, thus at this round, we have;

$$Y = Y_1 \quad [5.3]$$

It should be cleared why "Y" (or "Y_1") does not include "P_0". According to national income accounting rules, value of produced goods in the previous periods (i.e. last years) should not enter into the current period (i.e. current year) total value added accounts. Because, this value had been calculated and included in the previous years' income accounts.

In the second round, agent number one uses commodity C_1 to produce commodity C_2. He then sales it to agent number two with price of P_2 which is equal to P_1 (the price of C_1) plus some amount of value added (Y_2) that agent number one receives. Thus,

$$P_2 = P_1 + Y_2 \quad [5.4]$$

The required amount of nominal payment for this transaction is equal to P_2. Thus, we denote;

$$T_2 = P_2 \quad [5.5]$$

where T_2 stands for value of transaction of the second round. Total value of transactions (T) at this stage will be equal to sum of transactions values of the rounds one and two. That is;

$$T = T_1 + T_2 \quad [5.6]$$

Total value added at this stage is equal to,

$$Y = Y_1 + Y_2 \quad [5.7]$$

In the third round, agent number two uses commodity C_2 to produce commodity C_3. He then sales C_3 to agent number three with price of P_3. This new price (P_3) is equal to the price of P_2 plus his earned profit Y_3. Amount of Y_3 is equal to value added produced in this round. Thus;

$$P_3 = P_2 + Y_3 \tag{5.8}$$

The nominal payment for this transaction (T_3) is equal to P_3. That is;

$$T_3 = P_3 \tag{5.9}$$

Total value of transactions of the rounds of one, two and three is equal to sum of transaction values at different rounds. That is;

$$T = T_1 + T_2 + T_3 \tag{5.10}$$

Total produced value added will be equal to;

$$Y = Y_1 + Y_2 + Y_3 \tag{5.11}$$

Now, let us go to the J^{th} round. Similarly, we may state that agent number J-1 uses commodity C_{J-1} to produce commodity C_J. Then he sales C_J to agent number "J" with price of P_J. That is P_J is equal to the price of the commodity C_{J-1} (equal to P_{J-1}) plus the earned profit of agent number J-1 (equal to the amount of Y_J). Thus, we have;

$$P_J = P_{J-1} + Y_J \tag{5.12}$$

The nominal payment for this transaction is equal to P_J. Therefore;

$$T_J = P_J \tag{5.13}$$

Total value of transactions of all "J" rounds is equal to the sum of transaction values at different rounds. That is;

$$T = T_1 + T_2 + ... + T_J \tag{5.14}$$

Total produced value added in the economy will be equal to;

$$Y = Y_1 + Y_2 + \dots + Y_J \qquad [5.15]$$

Now, let us derive the relation between value of transactions (T) and total value added in the economy by solving [5.1] to [5.15]. It is clear that total value added in the economy at any round "J" is simply derived by [5.15]. This relation computes income in the national income accounting framework. So, total income at any round "J" is equal to;

$$Y = \sum_{j=1}^{J} Y_j \qquad [5.16]$$

Price of the commodity C_J at round "J" is simply derived by solving difference equation [5.12] with initial condition [5.1]. That is;

$$\begin{aligned}
P_1 &= P_0 + Y_1 \\
P_2 &= P_1 + Y_2 = P_0 + Y_1 + Y_2 \\
P_3 &= P_2 + Y_3 = P_0 + Y_1 + Y_2 + Y_3 \\
&\dots \\
&\dots \\
&\dots \\
P_J &= P_{J-1} + Y_J = P_0 + Y_1 + Y_2 + Y_3 + \dots + Y_J
\end{aligned} \qquad [5.17]$$

On the other hand we may write;

$$P_J = P_0 + \sum_{j=1}^{J} Y_j \qquad [5.18]$$

Total value of transactions at round "J" will be simply derived from [5.14], [5.13] and [5.12] as follows:

$$
\begin{aligned}
T = T_1 + T_2 + \dots + T_J = P_1 + P_2 + \dots + P_J = \; & P_0 + Y_1 + \\
& P_0 + Y_1 + Y_2 + \\
& P_0 + Y_1 + Y_2 + Y_3 + \\
& \;.\;\;.\;\;.\;\;. \\
& \;.\;\;.\;\;.\;\;. \\
& \;.\;\;.\;\;.\;\;.\;\; + \\
& P_0 + Y_1 + Y_2 + Y_3 + \dots + Y_J
\end{aligned}
\qquad [5.19]
$$

Therefore,

$$T = \sum_{j=1}^{J} T_j = \sum_{j=1}^{J} P_j = JP_0 + \sum_{j=1}^{J} \sum_{k=1}^{j} Y_k \qquad [5.20]$$

Different production stages and corresponding variables regarding our analysis all are depicted in table 5.1. At this stage, we should emphasize on some necessary points. Firstly, we are talking about nominal income and nominal value of transactions in this section and not real income and not volume of transactions. Secondly, we are also talking about price of a commodity as market value of that commodity. Therefore, the reader should not be confused with these terminologies.

Table 5.1

j	C_j	Y_j	Y	P_j	T_j	T
0	C_0	0	0	P_0	0	0
1	C_1	Y_1	Y_1	P_0+Y_1	P_0+Y_1	T_1
2	C_2	Y_2	Y_1+Y_2	$P_0+Y_1+Y_2$	$P_0+Y_1+Y_2$	T_1+T_2
3	C_3	Y_3	$Y_1+Y_2+Y_3$	$P_0+Y_1+Y_2+Y_3$	$P_0+Y_1+Y_2+Y_3$	$T_1+T_2+T_3$
. . .	. . .	. . .	. . .	. . .	. . .	. . .
j	C_J	Y_J	$Y_1+\dots+Y_J$	$P_0+Y_1+\dots+Y_J$	$P_0+Y_1+\dots+Y_J$	$T_1+\dots+T_J$

j : Production round.

C_j : Produced commodity at round j.

Y_j : Produced value added at round j.

Y : Cumulative value added.

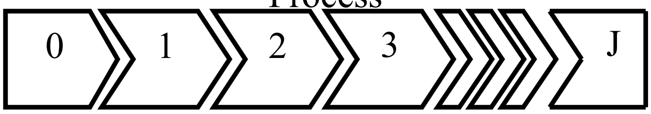
P_j : Price of commodity at round j.

T_j : Transaction value at round j.

T : Cumulative transaction value.

When we have only one initial commodity as initial input (C_0) and process of value added production is of integrated type (like our simple explanation), relation of transaction value and income can be simply shown by [5.20]. This integrated production process has a similar integration to figure 5.1.

Figure 5.1 Integrated Production Process



Since we are focusing on macro-frame of analysis, we can use a continuous forms of [5.16] through [5.20] relations. Because, we have many many commodities and production processes and finally only one "commodity" under the title of income or value added is produced, we may accept that in formation of total income, the amount of Y_j in [5.16] is very very small during the period of production, but their number (J) is very large relatively. However, this is actually highly realistic, since, on the average if we divide annual income of the economy to total number of seconds in a year we will see that the amount of value added of the economy produced in a second is very small (though total number of seconds in a year is very large). This leads us to use a continuous form of relation [5.16]. That is instead of discrete sum we can use integral sign. Let;

$$dY = \lim_{Y_j \longrightarrow 0} Y_j \qquad [5.21]$$

where d stands for differential. Note that Y_j was produced value added at round "j". This means that it is the difference of total value added at round "j" minus total value added at round j-1. So, in discrete case it is difference of total value added variable of two sequential round. Thus, we can easily adopt it as differential of the total value added in continuous case as [5.21]. In this regard, dY means very small changes of total value added.

Now, we can rewrite [5.16] by using [5.21] as;

$$Y = \int_0^J dY = J - 0 = J \quad ====> Y = J \qquad [5.22]$$

Now, consider the relation [5.18]. The variable P_J expresses the amount of money payment that one should pay to buy the final produced commodity C_J (at round J). From this payment, P_0 is the amount one pays and buys the commodity and no value added is produced by this purchase (transaction). The remaining amount of P_J is that amount of payment to buy commodity C_J that is equal to the total value added produced by production of C_J. This decomposition of transactions is very important when we will express our "exchange theory of money" in the proceeding sections.

However, the continuous form of [5.18] can be written as sum of these two components. That is;

$$P_Y = P_0 + Y = P_J = P_0 + J \qquad [5.23]$$

The third and fourth parts of [5.23] come from the result of [5.22].

In relation [5.20], the amount of JP_0 is the amount of transaction value, which does not produce value added. Let us denote this amount by T_0. In this regard, in a continuous frame we can use following procedure to get a similar equation to [5.20]. By [5.20] and [5.22] we have;

$$\int_0^Y P_Y dP_Y = \int_0^J P_Y dP_Y \qquad [5.24]$$

Replace [5.23] in [5.24] (and since $dP_0=0$), gives;

$$\int_0^Y (P_0+Y)d(P_0+Y) = \int_0^Y P_0 dY + \int_0^Y Y dY \qquad [5.25]$$

Thus we will have;

$$T = P_0Y + ½Y^2 = P_0J + ½J^2 \qquad [5.26]$$

as the continuous form of [5.20]. Note that in deriving [5.26], we can also use the following relation instead of the double sums on the right hand side of [5.20];

$$\int_0^Y \int_0^x dz dx = ½Y^2 \qquad [5.27]$$

where "Y", "x" and "z" replaced for "J", "j" and "k" as continuous form variables respectively. In [5.26] it should be noted that all variables are in values. Explicitly,

T : Total value of transactions.
Y : Total nominal value added (income).
P_0 : Total price (or value) of commodities produced in previous periods and are used as input in current period.

In [5.26], total value of transactions has been divided to two segments. One is that portion of transactions that does not produce value added. These transactions are equal to T_0 as;

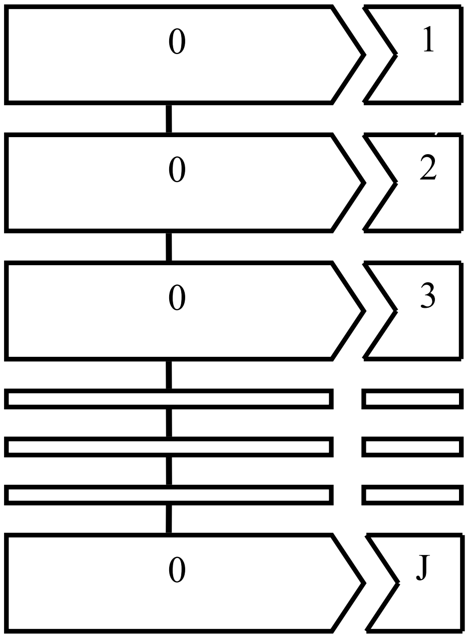
$$T_0 = P_0 Y = P_0 J \qquad [5.28]$$

The other segment is that portion of transactions that produce value added. Amount of these transactions is equal to half of the square of total value added in the economy. That is $\frac{1}{2}Y^2$.

5.1.2 Disintegrated Production Process

As we noted before in this process in contrast to integrated process, produced value added of a firm is not used as input for the other firm. This means that intermediate demands for commodities do not exist and all productions are used for final demand; in contrast to integrated process, which says final demand exists whenever we stop the production; and demands for commodities are of intermediate type. To understand the details of this opposite extreme process, we again go through the steps that value added is produced. The schematic shape of this type of process is shown by figure 5.2 below.

Figure 5.2 Disintegrated Production Process



Suppose that agent number zero has "J" units of C_0 commodity, having unique price of P_0. He sales these commodities to agents number 1,...,J with prices of $P_1,...,P_J$. So, in national income accounting we say commodities $C_1,...,C_J$ have been produced. Agent number zero's profit from each of these transactions is equal to $Y_1,...,Y_J$. The market prices and transaction values of these commodities are equal to;

$$
\begin{aligned}
&T_1 = P_1 = P_0 + Y_1 \\
&.\ \ .\ \ .\ \ . \\
&.\ \ .\ \ .\ \ . \\
&.\ \ .\ \ .\ \ . \\
&T_J = P_J = P_0 + Y_J
\end{aligned}
\qquad [5.29]
$$

where T_j is transaction value of j^{th} transaction. At this point, production of value added ceases. In compare to table 5.1, the table 5.2 can be considered for this production process. Total value added of the economy will be equal to;

$$Y = \sum_{j=1}^{J} Y_j \quad [5.30]$$

Total required nominal payments for all transactions will be equal to "T" as;

$$T = \sum_{j=1}^{J} T_j = \sum_{j=1}^{J} P_j = \sum_{j=1}^{J} P_0 + \sum_{j=1}^{J} Y_j = JP_0 + Y \quad [5.31]$$

Table 5.2

j	C_j	Y_j	Y	P_j	T_j	T
0	C_0	0	0	P_0	0	0
1	C_1	Y_1	Y_1	P_0+Y_1	P_0+Y_1	T_1
2	C_2	Y_2	Y_1+Y_2	P_0+Y_2	P_0+Y_2	T_1+T_2
3	C_3	Y_3	$Y_1+Y_2+Y_3$	P_0+Y_3	P_0+Y_3	$T_1+T_2+T_3$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
j	C_J	Y_J	$Y_1+...+Y_J$	P_0+Y_J	P_0+Y_J	$T_1+...+T_J$

j : Production round.

C_j: Produced commodity at round j.

Y_j: Produced value added at round j.

Y : Cumulative value added.

P_j: Price of commodity at round j.

T_j: Transaction value at round j.

T : Cumulative transaction value.

Now let us assume again that the amount of value added produced in each round is very small. Therefore again, definition of [5.21] prevails. By this assumption we may apply [5.22] again to [5.30] and we have total value of transactions for this production process as;

$$T= JP_0 + Y = YP_0 + Y \quad [5.32]$$

This equation has again, similar to [5.28], a part of transactions (equal to YP_0) that does not produce value added. The amount of T_0 should be exchanged (or transacted) until we can produce "Y" units of value added.

5.1.3 Mixed Production Process

Operationally, the two cited before extreme cases of integrated and disintegrated production processes both occur in economy. To combine these two processes, we use a convex combination of both. Let us use single prime (') and double prime (") symbols for integrated and disintegrated processes respectively; and symbols without prime for their convex combination. Total value of transactions as convex combination of both [5.26] and [5.32] will be equal to;

$$T=\alpha(P_0'Y+\tfrac{1}{2}Y^2)+(1-\alpha)(P_0''Y+Y) = [\alpha P_0'+(1-\alpha)P_0''+1-\alpha]Y+\tfrac{1}{2}\alpha Y^2 \quad [5.33]$$

where $a \in [0,1]$ is combination factor. Without loss of generality we may use P_0 as convex combination of P_0' and P_0''. That is;

$$P_0 = \alpha P_0'+ (1-\alpha)P_0'' \quad [5.34]$$

Thus, [5.33] can be written as;

$$T = (P_0 + 1 - \alpha)Y + \tfrac{1}{2}\alpha Y^2 \quad 0\leq \alpha \leq 1, P_0 \geq 0 \quad [5.35]$$

When α=1, equation [5.34] is the extreme case of integrated production process

(P_0=P_0') and when α=0, it explains the extreme case of disintegrated production process (P_0=P_0'').

In [5.35], value of those transactions that do not produce value added is again equal to [5.28].

5.1.4 Total Transaction and Foreign Trade

In previous section, we had assumed that trade is in balance and import is equal to export in the economy, so income will be equal to absorption. On the other hand our equation [5.35] implies for absorption as the following equation explains, where "A_n" stands for nominal absorption and "B_n" for nominal balance of trades;

$$T_A = (P_0+1-\alpha)A_n + \tfrac{1}{2}\alpha A_n^2 \quad B_n=0,\ 0\leq \alpha \leq 1,\ P_0 \geq 0 \qquad [5.36]$$

The left hand side variable T_A denotes transactions value due to absorption. Now assume that balance of trade is not equal to zero, and let us find the amount of total transactions due to foreign trade. In the case of exports, the amount of produced value added equal to value of exports is purchased by foreign countries from home country. There is a reverse case for imports, that is, the foreigner's produced value added is purchased by home country. Thus, total value of exports is equal to total value of transactions due to exports. Similar case occurs for imports. In transactions inside the boarder of a country, one is purchaser and one is seller. In transaction with foreign country, one purchases (imports) and one sells (exports) commodity. Total value added outflow is equal to exports and total value added inflow is equal to imports. Total transacted value with foreigners will be equal to net exports (exports minus imports). In this regard, we can write down the following equation,

$$T_B = Ex_n - Im_n \qquad [5.37]$$

Where T_B, Ex_n and Im_n denote transactions values due to foreign sector, nominal values of exports and imports respectively.

Total transaction in the economy will be sum of internal (T_A) and external (T_B) transactions as;

$$T = T_A + T_B \qquad [5.38]$$

Or, on the other hand we will have;

$$T = (P_0+1-\alpha)A_n + \tfrac{1}{2}\,\alpha A_n^2 + B_n \qquad 0 \leq \alpha \leq 1,\ P_0 \geq 0 \qquad [5.39]$$

This equation shows the relation of total transaction with absorption and balance of trade.

5.2 Quantity Theory of Money, Reformulated

Now, we are going to reformulate the quantity theory of money by using our important conclusion from the previous section given by equation [5.39]. For the sake of consistency through the text, keep in mind that in contrast to Fisher's notation, we use;

T: Total value of transactions of the economy in a given period.
t: Total volume of transactions of the economy in a given period.

In this regard, we have;

$$T = Pt \qquad [5.40]$$

Where "P" denotes general level of prices. Fisher's original quantity theory says;

$$MV = Pt = T \qquad [5.41]$$

Others' presentations of quantity theory explain;

$$MV = Py = Y \qquad [5.42]$$

Where "M", "V", "Y" and "y" were defined earlier. They propose that [5.42] is a good (but not complete) substitute for [5.41]. In "The purchasing power of money", Fisher explicitly derives [5.41] that total money required to handle all transactions multiplied by its velocity should be equal to value of transactions. Other revisionists tried to link total value of transactions to total nominal income in a loosed way and introduced [5.42]. In [5.42], the main problem or pitfall is the assumption that, they used real income as an exactly the same (scale) variable as volume of transactions. This was the main mistake that they undertook. A scale variable with coefficient one (y=1*t, or Y=1*T) is really a great specification error. In equation [5.35] we showed that total value of transactions actually has a parabolic relation with nominal income. Therefore, we can reformulate Fisher's quantity theory of [5.41] by using [5.39] as;

$$MV = (P_0 + 1 - \alpha)A_n + \tfrac{1}{2}\alpha A_n^2 + B_n = T = Pt \quad 0 \leq \alpha \leq 1,\ P_0 \geq 0 \qquad [5.43]$$

Simply, we may include the notion of general price level and real output (or output at constant price) by using the following simple identity (given constant foreign price);

$$Y = Py = P(A+B) = A_n + B_n \quad A_n=PA,\ B_n=PB \qquad [5.44]$$

Using this definitional identity in [5.43] we will have the following fundamental relation as reformulation of quantity theory of money;

$$MV = (P_0+1-\alpha)PA + \tfrac{1}{2}\alpha\,(PA)^2 + PB = T = Pt \quad 0 \leq \alpha \leq 1,\ P_0 \geq 0 \qquad [5.45]$$

This relation once relates money in circulation to total transactions and then relates total transactions to income components in the economy. It is interesting to note that in [5.45] if we use $\alpha=P_0=0$, we again will reach the equation of MV=Py as a special case of our formulation. In this case, by α=0 we mean the production process is disintegrated and by P_0=0 we mean no initial valued input is used in the process of value added production.

5.3 Money Demand and Different Motives

Let us now return to our previous discussions about money demand and different motives of transaction, speculation and precautionary. Up to here, we found the main relation between income and total transactions. That is the right hand side of quantity theory of money should be replaced by $(P_0+1-\alpha)PA+\frac{1}{2}\alpha(PA)^2+PB$ instead of Py. For example, equation [2.12] and [2.13], which includes different money demand motives should be rewritten as;

$$V\{(1-r)[M_T+(1+i)M_S]+M_P\} = (P_0+1-\alpha)PA + \frac{1}{2}\alpha\,(PA)^2 + PB \qquad [5.46]$$

where M_P is the same as [2.13]. Equation [2.14] will simply become;

$$V[M_T+(1+i)M_S] = (P_0+1-a)PA + \frac{1}{2}a(PA)^2 + PB \qquad [5.47]$$

All of our assumptions and discussions about velocities, induced and autonomous speculative demands and so on remain unchanged. However, total demand equation is simply derived from [5.45] as;

$$M = (P_0+1-a)PA/V + \frac{1}{2}a(PA)^2/V + PB/V = T/V = Pt/V \qquad [5.48]$$

5.4 Empirical Analysis

Let us now test our important proposition of [5.35] empirically for the data of United States. Thus, we are going to test the relation between total value of transactions and nominal income as specified by the following regression equation.

$$T_t = ß_1 Y_t + ß_2 Y^2{}_t + u_t \qquad [5.49]$$

where;

T_t : Total value of transactions at time t.

Y_t : Nominal income at time t.

u_t : Disturbance term.

$ß_1$,$ß_2$: Regression coefficients.

For the sake of simplicity, in this model we assumed that PB is a component of random error term obeying classical assumptions of least squares regression. This assumption does not affect our results so much. According to this specification, the estimated values of $ß_1$ and $ß_2$ should have the following restrictions.

$$ß_1 = P_0 + 1 - \alpha \geq 0 \qquad [5.50]$$

$$0 \leq ß_2 = \alpha/2 \leq ½ \qquad [5.51]$$

Another test is to be performed is equation [5.49] accompanying with intercept term $ß_0$. That is;

$$T_t = ß_0 + ß_1 Y_t + ß_2 Y^2{}_t + u_t \qquad [5.52]$$

If our proposition is true we should reach significant $ß_1$ and $ß_2$ in [5.49] and [5.52] within the interval given by [5.50] and [5.51] and insignificant $ß_0$ in [5.52].

Since transaction data is not available, similar to previous researches, we try to employ total debits data and its modifications as proxies for total value of transactions. The debit data captures the values of initial, intermediate and final transactions. Therefore, according to our previous discussions, debits data will be more consistent with what we mean by transaction.

To calculate total debits, we combined debits on demand deposits as a proxy for total deposit transactions and three different types of debits on currency as proxies of total currency transactions. To account for the level of currency transactions we follow the assumptions and data applied by Komijani (1983). In this regard, three alternative scenarios adopted to approximate total debits, namely, $T^{(1)}$, $T^{(2)}$ and $T^{(3)}$ with following definitions;

$T^{(1)}$ = Debits on demand deposits in all commercial banks.

$T^{(2)}$ = Debits on demand deposits in all commercial banks + 15 × (Stock of currency).

$T^{(3)}$ = Debits on demand deposits in all commercial banks + The dollar value of the amount of currency "received and counted" by Federal Reserve System.

To test the equations [5.49] and [5.52] with above three proxies for total value of transactions, we used the data provided by Komijani (1983) for the period of 1952-1980 for the United States of America. Cochrane-Orcutt procedure of estimation applied to the models [5.49] and [5.52]. The results of calculations are depicted in table 5.3.

The first three rows are corresponded to equations [5.49] and completely confirm our hypothesis and model specification with special attention on the conditions of [5.50] and [5.51]. The rows of four through six of the table are corresponded to the model [5.52]. These rows also confirm our hypothesis that the estimated intercept should be insignificant. All other calculated statistics confirm our hypothesis strongly.

Table 5.3

No	Dep. Var.	$ß_0$ ($Sß_0$^)	$ß_1$ ($S_{ß1}$^)	$ß_2$ ($S_{ß2}$^)	ρ (S_ρ)	R^2	Durbin Watson
1	$T_t^{(1)}$	--- ---	2.3236 (0.6327)	0.0080 (0.0003)	0.3572 (0.2228)	0.997	1.3708
2	$T_t^{(2)}$	--- ---	3.1624 (0.6357)	0.0079 (0.0003)	0.3514 (0.2258)	0.997	1.3769
3	$T_t^{(3)}$	--- ---	2.3793 (0.6328)	0.0080 (0.0003)	0.3562 (0.2233)	0.997	1.3710
4	$T_t^{(1)}$	-80.416* (1069.66)	2.4620 (1.9902)	0.0080 (0.0007)	0.3509 (0.2505)	0.997	1.3747
5	$T_t^{(2)}$	+85.912* (1080.53)	3.0158 (2.0054)	0.0080 (0.0007)	0.3568 (0.2467)	0.997	1.3736
6	$T_t^{(3)}$	-61.996* (1071.71)	2.4855 (1.9939)	0.0080 (0.0007)	0.3515 (0.2504)	0.997	1.3739

* Insignificant

5.5 Reformulated Quantity Theory of Money

Now let's enhance our reformulated quantity theory of money given by [5.45] to included exchange rate component. By this enhancement, we try to explain how exchange rate is determined when we use Fisher's original quantity theory implication of the relation between money and transaction. That is we try to use our explanation of money-transaction-income process to describe how exchange rate is determined. In previous sections of external and international monetary equilibriums, we used a special case to introduce the exchange rate in the model. The assumption behind the discussions in those sections was the equality of nominal transaction and nominal income as is evident from equation [2.1] , [3.8], [4.3] and [4.4]. In the mentioned sections, we assumed that nominal transaction due to absorption is equal to total nominal absorption. That is we has assumed "$PA=T_A$". At this stage, we have a specific equation to express nominal transaction due to absorption given by [5.36]. To generalize the discussions of cited sections we should replace "PA" in the equations stated in those sections by "T_A" expression given by [5.36]. According to this point, we may rewrite the equations of [3.8] through [4.71]. But, in order to express as short as possible we only highlight the main equations and conclusions and avoid to present the corresponding derivations. At the first stage let's rewrite [5.38] as;

$$Pt = Pt_A + Pt_B \tag{5.53}$$

where,

t = Total volume of transactions

t_A = Volume of transactions due to absorption

t_B = Volume of transactions due to balance of trade

Note that according to our previous discussion we have;

$$t_B = B \tag{5.54}$$

and

$$B_n = PB = Pt_B = ePb \tag{5.55}$$

Similar to [3.8] by [5.44] and [5.53] and [5.55] we have,

$$V(M+en) = PT = P(t_A+t_B) = Pt_A + ePb = T_A + ePb = (P_0+1-a)PA + ½a(PA)^2 + ePb = P(t_A+eb) \tag{5.56}$$

Relations [3.11] through [3.20] all remains unchanged except for [3.16] that the equation holds for "t_A" instead of "A". Similar to equation [3.25] in this case we have,

$$\hat{}e = \hat{}(VM-Pt_A) - \hat{}(Vn-Pb) = \hat{}(VM-T_A) - \hat{}(Vn-B) \quad [5.57]$$

which means rate of change of exchange rate is equal to difference of two rates of changes in internal and external sectors imbalances in money and commodity markets. On the other hand, similar to [3.26] we may write;

$$e = \frac{Pt_A-VM}{Vn-Pb} = \frac{T_A-VM}{Vn-B} = \frac{(P_0+1-\alpha)A_n+½\,\alpha\, A_n{}^2-VM}{Vn-B} \quad [5.58]$$

which means that exchange rate is the ratio of internal and external imbalances between money value and transaction value. Equation [3.33] will have the following revised form,

$$V[(M_T+en_T)+(1+i)(M_S+en_S)]=(P_0+1-\alpha)PA+½\alpha(PA)^2+ePb \quad [5.59]$$

This equation shows the equilibrium condition of money and commodity markets in our reformulated form of quantity theory of money when we have foreign trade and foreign currency in the economy accompanying with domestic money and commodity markets and a unique interest rate.

The above revisions can be simply done in the international monetary equilibrium discussions given by equations [4.1] through [4.71]. The reader may do these reformulation by replacing "PA" in all these equations by $(P_0+1-\alpha)PA+½\alpha(PA)^2$ for home and foreign superscripts. The results and conclusions do not change actually, but we do not derive them at this stage.

Chapter Six

Exchange Theory of Money

6.1 Philosophical and Logical Notes

There are many definitions for money in the literature and as was stated in the introduction chapter it is very controversial to accept a unique definition for money. Money as store of value or medium of exchange are not wide-spectrum definitions. If money is an economic commodity, it should have its own price, if we do not accept a price for it we should never talk about its demand! Because, economically, demand for everything occurs when a price comes into account – except free goods, which are not economically considered. This is an economic law and not an interpretation. When money is used as store of value, it cannot be neutral and abolition of neutrality makes money an economic commodity. If money is used as medium of exchange, it works like a lubricator (or catalyzer) in the economy. A catalyzer should never enter into the operation directly. On the other hand, a catalyzer just facilitates the operation and should not change the nature of operation itself. In this regard, if money works as catalyzer it should not have demand in the economy; because it is not a commodity. Therefore, money cannot be defined as medium of exchange solely.

This discussion leads us to loose the neutrality of money as has been discussed in the literature and think about it (money) as an economic good. The most obvious characteristic of an economic good is its price, so we should look for the price of money. Some believes that rate of interest is price of money! It should be said that rate of interest can be rate of return of "money capital" not money itself. "Money capital" is the money that we have invested it and is capital now, not money.

6.2 Net Velocity of Circulation of Money and Income

In the chapter two of this book, we tried to interpret the velocity of circulation of money and also the rate of interest plus one as price of money for transaction and speculative moneys. Now, we should clarify some important points that quantity theorists (except Fisher) have underestimated. First, look at the Cambridge quantity theory of money equation [2.1]. The product "Py" on the right hand side of this equation is equal to total value added in the economy. This product (or account) measures the expenditure or income stream that comes from current production of goods and services. Transactions or speculations that have zero value added or transfer ownership of existing assets are in general, not reflected in this account, because they do not involve in current value added production – in national accounting framework. On the other hand, transaction involving purely as asset exchange has no direct effect on current production and thus does not enter into national income accounts. But there should be some exchanged money for these transactions! In the left hand side of equation [2.1] we computed the velocity of circulation of money "V" in such a way that if we multiply it by money stock "M", the resulting value be equal to "Py" (or total value added in the economy). But while we have ignored the zero value added transactions in the right hand side, we also ignore the necessary money value corresponding to these transactions (at the left hand side). Therefore, velocity of circulation of money should be higher than what is used as "V" in [2.1]. This means that fixed stock of money should circulate faster to provide all of transactions with non-zero and zero values added. Thus, we rename the famous velocity of circulation of money (given by [2.1]) to net velocity of circulation of money, because it does not account the money circulation for these transactions that involve purely as asset exchange and have no direct effect on current production. Let us redefine the net velocity of circulation of money as the times that an average unit of money circulates to produce value added in a specific period. With this definition we may draw a borderline between commodity (or other assets) and money. Now put the total "money" stock and total value added in the economy for a specific period in a bundle and name it as "exchangeable assets" bundle. At the first glance, there is no distinction between the two components of this bundle. All of the elements in this bundle are exchangeable. We may draw a line to discriminate money and commodity in this bundle. If an element is exchanged by another element and makes value added a commodity has been produced (and exchanged) and if this exchange does not make value added money has been exchanged. This point will be used for defining money later. In this regard, the main difference between money and commodity arises in procuring and not procuring value added; since, money and commodity both are exchangeable with each other and within themselves. Note that in the domain of exchangeable things we put only commodity that is produced in a specific period and the other thing as "money" which we will define it later. Concentrate on the following reasoning;

$\vee \equiv$ "Or" operator.

$C \equiv$ Commodity set.

$M \equiv$ Money set.

$C \cup M$ = Exchangeable assets (union of C and M).

If a$\in$ (CUM) is exchanged with b$\in$(CUM) and a$\cap$b=ø and value added is produced ===> (a$\vee$b) $\in C$; if a$\in C$ ===> b$\in M$;

if a$\in M$ ===> b$\in C$.

If a facilitates production of value added ===> a$\in M$ ===> b$\in C$. ===> net velocity of circulation of a is greater than net velocity of circulation of b.

If b facilitates production of value added ===> b$\in M$ ===> a$\in C$. ===> net velocity of circulation of b is greater than net velocity of circulation of a.

If a$\in$(CUM) is exchanged with b$\in$ (CUM) and a$\cap$b=ø and value added is not produced ===> ((aUb) $\in C$) $\vee$ ((aUb) $\in M$).

This reasoning expresses that money exist when value added is and is not produced in an exchange; and commodity is produced and exchanged whenever value added is produced; and money can be distinguished from commodity in their relative velocities of circulations.

Now let us go through the "exchangeable assets" bundle and make some discussion by means of quantity theory. Returning to equation [2.1], one may redefine the price index "P" as velocity of circulation of value added. On the other hand, price can be regarded as the times that on average, one unit of value added is produced (which means exchanged). This creation of value added is similar to circulation of value added in the process of production in the economy. Therefore, as money circulates in the economy, value added circulates too. Thus, we may think of "P" as velocity of circulation of commodity. Given this exposition, two components of money and commodity in our "exchangeable assets" bundle both are circulating.

Commodity circulates and makes added value and money exchanges with money and does not make added value. Rewriting the Cambridge quantity theory as;

$$\frac{y}{M} = \frac{V}{P} \qquad [6.1]$$

We may expose that the ratio of income to money is equal to the inverse of their prices or inverse of their velocities of circulation. That is, if quantity of output produced per unit of money (y/M) increases, the velocity of circulation of money per one time circulation of value added should increase, or inversely velocity of circulation of commodity per one time circulation of money should decrease to compensate the primary increase in y/M. This discussion shows the relationship of money and commodity within the exchangeable assets bundle when we undermine net velocities of circulation of money and income. Net velocity of circulation of income is defined similar to net velocity of circulation of money. Velocity of circulation of commodity (or income) is the times that an average unit of value added (or commodity or income) circulates to produce new value added in a specific period. Thus, any exchange of commodity with commodity that does not make value added, should not come into computation of net velocity of circulation of income. This restriction is completely consistent with our interpretation of price (P) as velocity of circulation of income. Because if we increase the total transactions of commodities that do not produce value added (which means price is constant) it means that we have shifted both demand and supply curves to the right which their new intersection has old equilibrium price and new larger equilibrium quantity.

6.3 "Exchange Theory of Money"

Now let us develop the quantity theory of money to a more general case namely "exchange theory of money". In the previous sections, we discussed that why we should rename the velocity of circulation of money to the net velocity of circulation of money and also we noted the velocity of circulation of income or commodity. Now let us go further and raise the notion of barter velocities of circulation of money and commodity (income). We may classify all money and commodity exchangeable cases in the following table 6.1.

Table 6.1

Case	Economic Implication
Money with money at a point of time	Economically nonsense
Commodity with Commodity	Economically nonsense in macro frame because we have only one commodity at macro level (value added). That is, exchange of previously produced value added with itself.
Money with commodity	"Economic exchange" (when value added is produced).
	"Barter exchange" (when value added is not produced). That is, some value of money is exchanged with the same value of commodity and no surplus of income or value added is produced in this transaction.

It is obvious from table 6.1, when money is exchanged with money (at a point of time and not in form of time-based loan and borrow) nothing happens and there is no economic implication for this case. On the other hand, nobody makes this exchange because there is no profit (value added) in this transaction. When commodity is exchanged with commodity it means equal amount of value added is exchanged with the same amount and nothing happens again at macro level of analysis; since at this level we have only one type of commodity which is value added, and we have not made distinctions among commodities as we observe at micro level of analysis. When commodity is exchanged with money, two cases arise. First, "economic exchange" occurs when this exchange causes value added to be produced and the second "barter exchange" occurs when this exchange does not produce value added. Note that we use the term "barter" versus "economic" implying no value added is produced by a "barter" exchange; and value added or income is produced by an "economic" exchange. The Cambridge quantity theory of money in form of equation [2.1] just concentrates on "economic exchange", because at the right hand side of equation [2.1] we can only concentrate on income or "economic exchange" context. But, we may develop the "exchange theory of money" to include barter and economic exchanges both with regarding the notions of net velocities of circulation of money and income. Now look at the equation [6.2] with explanations on it.

Barter velocity of circulation of money

Net velocity of circulation of money

Price or net velocity of circulation of income

$$M \left(V^{M}_{b} + V^{M}_{n}\right) = \left(P + V^{y}_{b}\right) y \qquad [6.2]$$

Stock of money

Income or commodity

Barter velocity of circulation of income

In this equation, both barter and economic exchanges have been entered simultaneously. The variables V^{M}_{b} and V^{y}_{b} are barter velocities of circulation of money and commodity respectively with the following definitions. V^{M}_{b} is the times that an average unit of money is exchanged with commodity ("barterly" not "economically") and makes no value added. V^{y}_{b} is the times that an average unit of value added is exchanged with money ("barterly" not "economically") and value added is not produced. V^{M}_{n} and V^{y}_{n} are the net velocities of circulation of money and commodity as defined earlier. If we denote the gross velocities of circulation of money and income by V^{M}_{g} and V^{y}_{g} respectively, we have the following identities;

$$V^{M}_{g} = V^{M}_{b} + V^{M}_{n} \qquad [6.3]$$

$$V^{y}_{g} = P + V^{y}_{b} = V^{y}_{n} + V^{y}_{b} \qquad [6.4]$$

The equation [6.2] can be written as follows,

$$MV^{M}_{g} = yV^{y}_{g} \qquad [6.5]$$

That is money multiplied by its gross velocity of circulation is equal to income multiplied by its gross velocity. Name the relation [6.2] or [6.5] as "exchange theory of money" which encounters both "barter" and "economic" exchanges.

To clarify the relation of the Cambridge quantity theory of money and exchange theory of money look at the relation [6.6] with its corresponding notes.

$$MV^{M}_{b} + \boxed{MV^{M}_{n} = Py} + yV^{y}_{b} \qquad [6.6]$$

Cambridge quantity theory of money

Exchange theory of money

6.4 Exchange Theory of Money and Reformulated Quantity Theory

Let us include our reformulation of quantity theory given by [5.45]. We may rewrite [5.45] as;

$$MV = P_0Py + Py + \tfrac{1}{2}\alpha P^2y^2 - \alpha Py \qquad [6.7]$$

Without loss of generality, we can simply adopt the left hand side of [6.6] instead of MV in [6.7]. That is;

$$MV^M_b + MV^M_n = Py + P_0Py + \alpha\,(\tfrac{1}{2}P^2y^2 - Py) \qquad [6.8]$$

Now compare [6.8] with [6.6]. The left hand sides of both equations are equal. The first terms of the right hand sides of both [6.8] and [6.6] equations are again equal. This is true also for the second terms of the right hand sides in definitional aspect of barter velocity of income (V^y_b) and those transactions that do not produce value added from $T_0=P_0Py$ of [5.28]. In this regard, barter velocity of income is equal to P_0P. That is;

$$V^y_b = P_0P \qquad [6.9]$$

And thus;

$$T_0 = yV^y_b = P_0Py \qquad [6.10]$$

Let us now check the last parentheses of [6.8]. Remember that "α" was the criterion of integration of production in the economy. When $\alpha=0$, we have a disintegrated production structure and when $\alpha=1$, the structure of production of the economy is completely integrated. If $\alpha=0$ in [6.8], the exchange theory of money, when we use reformulated quantity theory, or quantity theory of money itself, does not change. But when "α" is not equal to zero, the effect of integrity of production structure comes into account. When "α" increases from zero to one, the effect of integrity also increases and causes the reduction of "Py" term effect in equation and increments the $\tfrac{1}{2}P^2y^2$ term effect. However, in the case of reformulated quantity theory, our exchange theory of money is depicted by equation [6.11].

$$MV^{M}_{b} + MV^{M}_{n} = Py + P_{0}Py + \alpha\left(\tfrac{1}{2}P^{2}y^{2} - Py\right) \qquad [6.11]$$

Cambridge quantity theory of money

Exchange theory of money

Reformulated exchange theory of money

6.5 Aggregate Supply and Demand and Total Transactions

When the first digression occurred in Fisher's quantity theory of money, actually the logic of this theory ignored. Fisher in his book "Purchasing Power of Money" explicitly enters transactions at right hand side of quantity theory equation. He even declares his finding similar to the physical law of balance as is depicted by the following picture:

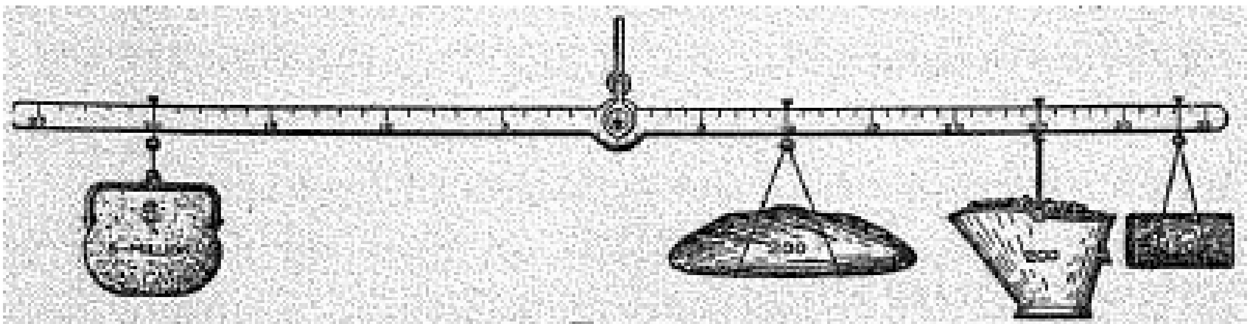


FIG. 2.

Fisher, Irving, (1911) The Purchasing Power of Money: Its Determination and Relation to Credit, Interest, and Crises. Reprints of Economic Classics. New York.

In Fisher original quantity theory we have:

$$MV = T = P\,t \qquad [6.12]$$

where;

M: Supply (or demand for) money.

V: Number of times an average unit of money changes hands (velocity of circulation of money).

P: Price level.

t: Quantity of goods and services transacted.

T: Nominal total transactions.

If we compare this original specification of Fisher's quantity theory to what is calculated in System of National Accounts (SNA) we would conclude that:

Total nominal transaction is equal to aggregate demand at current prices and is equal to aggregate supply at current prices.

This main conclusion is because the fact that total output is equal to gross product of the economy plus intermediate goods. We do not enter into the details of accounts of SNA here and interested readers may see SNA documents. The mathematical relations among macro variables at national income accounts can also be seen in mathematical frame of Macro-econometric Model of Iran.[2]

With this understanding, let us revise [6.2] as

$$M\,(V^{M}_{b} + V^{M}_{n}) = Py + V^{y}_{b}\ \mathrm{int} \qquad [6.13]$$

Where "int" stands for intermediate input at national level at constant prices. Therefore, we return back to original Fisher quantity theory of money, but with some clarifications that:

[2] Bijan Bidabad, Macroeconometric Model of Iran, version 6.1, technical document. Lap Lambert Academic Publishing, OmniScriptum GmbH & Co. KG, ISBN: 978-3-659-14252-9, Winter 2014.

$$MV = T = P\,(y + int) = P \,.\, as = P \,.\, ad = AS = AD \qquad [6.14]$$

In this equation, since P is an implicit price deflator for aggregate supply or demand we used it instead of P and $V^{y}{}_{b}$ both. AS and AD stand for aggregate supply and demand at current prices and "as" and "ad" for aggregate supply and demand at constant prices.

6.6 Income and Intermediate Inputs

By comparing the discussion in previous section with [5.35] we will reach to an interesting relation between total income and intermediate goods. That is,

$$T = (P_0 + 1 - \alpha)Y + \tfrac{1}{2}\alpha Y^2 = P\,(y + int) = Y + INT = AS = AD \qquad 0 \le \alpha \le 1,\ P_0 \ge 0 \qquad [6.15]$$

Where INT stands for nominal intermediate goods. The following result comes from manipulation of the above relation:

$$INT = (P_0 - \alpha)Y + \tfrac{1}{2}\alpha Y^2 \qquad 0 \le \alpha \le 1,\ P_0 \ge 0 \qquad [6.16]$$

This equation shows the relation between nominal intermediate goods and total value added in the economy.

6.7 Money Definition Conclusion

We should first defined money in the first pages of this book, but we regret to bring it here because of its relevancy. As it was discussed in the previous pages, intrinsic disability of money to make value added is its main characteristic. Money should have price, because it has demand and supply and it is an economic good. Money is used as medium of exchange and money is generally acceptable. It was noted in previous sections, money and commodity are both exchangeable, but the question is: which one of them is medium of exchange for other in our exchangeable assets bundle? In our defined bundles, we just referred to existence of different items that can be exchanged with other one. Some of these exchanges were barter and others were economic. However, we notified that money should not produce value added but can measure the volume of produced value added. In this regard among all kinds of assets in our bundle which one can be used to measure the volume of produced value added and which one can be used as medium of exchange for the others. We concluded that if money is used for measuring or intermediacy, it should have the fastest velocity of circulation among all assets. When this velocity is fastest, it means that money is generally acceptable and it can be used as means of payment and it is not inherently productive and does not make value added intrinsically. With these qualifications, money can be defined as anything when it is exchanged, intrinsically it does not make value added and inherently it is not productive and it has the fastest velocity of circulation among all assets. Thus, it becomes generally acceptable as means of payment or in final settlement of a debt and also as a measure of value.

6.8 Degree of Moneyness

When we assign the fastest velocity of circulation to money, the acceptability of public is at behind of our mind and intermediacy and being criterion of measuring the economic activities are all accepted as properties of money. If we find an item that has the highest acceptability and being criterion of measuring it will have the fastest velocity of circulation in the economy and on the other hand, it has the highest degree of moneyness. So, velocity of circulation of any asset can be regarded to its degree of moneyness. This criterion for measuring the degree of moneyness can be used for measuring the speed of affecting the monetary policies in the economy.

With this expression, we may check the components of different assets to conclude that are they acceptable as money or not.

Resources

There are many books and papers on this broad discussion; the reader's attention is turned just to the following references. Some of references are collections of original papers.

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General Monetary Equilibrium

We are going to examine the bases of monetarism. We investigate the philosophical bases of equilibrium in money and commodity markets and try to reinterpret "quantity theory of money" to express domestic monetary equilibrium. We deal with important monetary variables as velocity of circulation of money, rate of interest, supply and demand for money with emphasis on different demand motives. External monetary equilibrium with foreign money and exchange rate and price level determination in an open economy are discussed. The subject is generalized to international monetary equilibrium. Important monetary rules that determine exchange rate at international level are introduced. Relation of interest rate and exchange rate is also discussed and determined. Moreover, we reinterpret and develop the original Fisher's quantity theory of money by introducing quantitative relation between transaction and income. Empirical investigations all confirm our formulation. Finally, we define money logically and philosophically and the relation among transaction, output, intermediate input, aggregate supply and demand and their relations to money are explained.

Bijan Bidabad

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