

Nonlinear Impact of Interest Rates on Price Level (Smooth Transition Autoregressive Approach)¹

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Abstract

Economic and monetary policy-making shall benefit from the nominal interest rates and inflation causal relationship for macroeconomic issues. In this paper, by expanding Fisher's Quantity Theory of Money, we investigate this relationship using the "Smooth Transition Autoregressive" approach for Iran's economy.

The results show that when the interest rate rises (particularly when it exceeds the threshold), its positive impact on inflation will increase. Moreover, the effect of a low-interest rate increase does not have a severe effect on price change. It is concluded that interest rate is not a good policy instrument for high inflation dampening, and those policies that diminish money circulation velocity might be more effective for general price level control. Macro-economically, the results show that in the long run, interest rate yield that is paid to the factors of production is practically eliminated by reducing the purchasing power of money due to the price increase in the economy.

This might be understood as one of the pearls of wisdom that Almighty God has cited in the Holy Qur'an about the prohibition of usury: "**God effaces usury**"⁵ means that Almighty God eliminates the effects of the usury yield increment in the economy.

Keywords: Interest rate, Price, Smooth Transfer Autoregressive Model, Quantity Theory of Money, Irving Fisher

JEL: E31, E43, E40

Introduction

The primary variable that is affected by the general price level is the real money balance. In other words, as the price increases, the real money supply decreases as well. In the Keynesian analytical framework, a decline in the real money supply (an excess demand for money) pushes the economy into a new equilibrium. Generally, according to Walras equilibrium, to balance the economy, an excess of money demand in the money market leads to a surplus of supply of bonds in the bonds market. Therefore, it is theoretically expected that by increasing the prices, the interest rate will also increase. In this regard, there is a positive causal relationship between inflation to nominal interest rates theoretically. In other words, an increase in inflation will increase interest rates in the economy.

¹- This paper is an excerpt from the first author's Ph.D. dissertation at Islamic Azad University, Qazvin Branch, Iran.

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⁵- Sura: Al-Baqarah, verse: 276. **يَمْحَقُ اللَّهُ الرِّبَا**

How the interest rate affects inflation can be explained in different ways. One of the mechanisms of influencing the interest rate on inflation is the cost of using capital, so raising the interest rate increases the cost of using capital, which ultimately leads to increased markup prices and production costs.

Increasing production or transfer costs to the left of the supply curve of the whole economy will ultimately increase inflation.⁶ Also, changes in interest rates affect inflation by affecting the money supply, so in endogenous models of money, where the money supply is a direct function of interest rates, the money supply increases as interest rates increase. According to the quantity theory of money, increasing the money supply in the long run and the short run will increase the general level of prices. Although the money supply may not have a significant effect on inflation in a broad recession, in normal circumstances, and at least in the medium term or long term, the effect of the money supply on inflation is positive and significant.

Therefore, theoretically, an increase in interest rates is expected to increase the price level, and therefore it is argued that there is a possibility of a causal relationship between interest rates and inflation. Another mechanism for explaining the relationship between interest rates and inflation is the well-known relationship between nominal and real interest rates, which has a long history in the economic literature. In general, the relationship between real and nominal interest rates indicates a positive relationship between inflation and nominal interest rates. This was introduced by William Douglas before the 1840s, and Henry Thornton used the idea to explain the relationship between real and nominal interest rates. Alfred Marshall (1890) examines the relationship between the nominal rate and the inflation rate as follows:

$$r = i - \text{inf} - \text{np} \quad (1)$$

In the above relation, r_t is the real interest rate, and i_t is the nominal interest rate, inf is the inflation rate, and np is the cross effect of the two variables, the nominal interest rate and the inflation rate. Therefore, from Marshall's point of view, nominal interest rates and inflation rate are directly related to each other.

Unlike Marshall, John Baits Clark (1895) believes that real interest rates are fixed, and in his studies, he examined the effect of inflation on nominal interest rates. In his view, the nominal interest rate changes in proportion to the inflation rate. In other words, the interest rate is directly related to the inflation rate, and if it decreases (increases) by 2%, the nominal interest rate must also decrease (increase) by 2%.

However, the relationship between nominal and real interest rates did not have a good analytical framework until Irving Fisher. Irving Fisher (1896) used the studies of others to coherently explain the theory of inflation and interest. The introduced Fisher relation is as follows:

$$r = i - \pi^e \quad (2)$$

Where π^e is the expected inflation rate. With this expression, it can be said that theoretically, the relationship between nominal interest rates and the inflation rate is positive, and there is a two-way causal relationship⁷ between these two variables.

⁶ Bidabad, Bijan; Kalbasi Anaraki. Nahid. (2019). Inflation targeting in Iran. *Asian Finance & Banking Review*, 3(1), 74-88.

<http://www.bidabad.ir/doc/Inflation-Targeting-in-Iran.pdf>

<https://www.cribfb.com/journal/index.php/asfbr/article/view/296>

⁷ Bidabad, Bijan (2019) Triangular Causality in Controlling Parallel Exchange Market by Monetary Targeting and Anti-Inflationary Policies. *Australian Finance & Banking Review*, 3(1), 20-32.

<http://www.bidabad.ir/doc/triangular-causality-en.pdf>

<https://www.cribfb.com/journal/index.php/afbr/article/view/299>

<http://www.bidabad.ir/doc/triangular-causality-en.ppt>

The Quantity Theory of Money

The quantity theory of money dates back to the sixteenth century when the influx of gold from the colonies caused some economic disruption. At that time, the price level in Spain increased tenfold, and as a result, the price of wheat in France almost tripled. The Paris Court of Audit, together with a lawyer named Jean, after an extensive investigation, concluded that the main reason for the general rise in prices, or inflation, was the influx of precious metals from new lands into Spain and France, and for the first time, the relationship between inflation and the money supply was identified. But the first person who formulated this theory after about three centuries was the English economist David Ricardo:

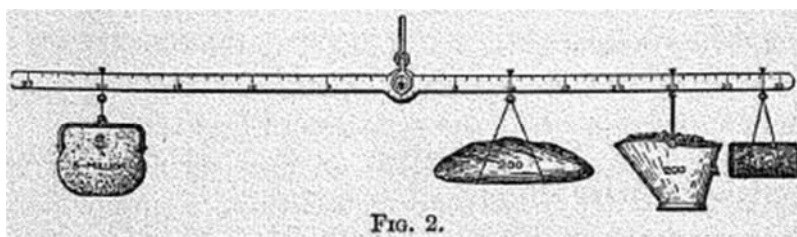
$$M = P \cdot t \quad (3)$$

In which M is the volume of money, P is the general price level, and T is the volume of transactions, or indeed the volume of nominal production.

Thirty years after Ricardo, John Stuart Mill added a parameter called the velocity of money to the above relation and gave the relation $M \cdot V = P \cdot t$. Until the early twentieth century, M meant only banknotes and coins in circulation. But early in the last century, the American economist Irving Fisher also introduced quasi-money into the quantity theory of money. For this reason, some economists also refer to this relation as Fisher's relation. Fisher's quantity theory of money is the most important theory regarding the simultaneous equilibrium of money and commodity markets. By a simple derived example from physics, Fisher⁸ showed that there is always the following linear relationship between money and price:

$$\text{Price of goods} \times \text{Quantity of transacted goods} = \text{Money volume} \times \text{Money circulation velocity}$$

He uses scales to explain this theory, and according to the mechanical relations between the items of the variables of the above equation, it should be said that while everything is fixed (*Ceteris Paribus*), the above mathematical relation is always established, and the above relation as like as the law of scales in physics, is a law in economics.



Picture from the book *The Purchasing Power of Money* by Irving Fisher

$$M \cdot V = P \cdot t \quad (4)$$

Where M is the volume of money or money supply; V is the velocity of circulation of money (the number of times a unit of currency is used for the transaction in a given period), and t is the amount of transacted goods during the specified period, and P is the average price of the transaction. This relationship states that the total cost of purchasing goods and services, i.e., T, is equal to the total money that sellers earn from goods and services, i.e., P.t. Money multiplied by the velocity of money actually provides the transactors with commodity transactions due to money payments. Thus, the money paid for goods and services must be equal to the amount of goods and services purchased.

Post-Fisher economists, like the Cambridge school, always used another alternative variable instead of transaction value due to the lack of statistical information. For example, they used Gross National Product

⁸ - Irving Fisher (1911). *The Purchasing Power of Money, Its Determination and Relation to Credit, Interest, and Crises*. MacMillan Pub.

instead of an approximation for the value of transactions, which caused various problems for the studies. It can be shown that the relationship between GDP and transaction value is not definite⁹. The current standard of the UN System of National Accounts (SNA) considers all transactions that can take place in the economy, and as a result, this system can be used to help solve this problem. In other words, in the SNA 1993, without this issue being emphasized in detail, it can be seen that the nominal volume of transactions in an economy is equal to the aggregate nominal supply in the national accounts framework. And the aggregate supply is derived from the total value added of different sectors plus the value of intermediate goods. That is:

Total value of transactions in the economy = Aggregate nominal supply in the economy

This is primarily due to the comprehensiveness of the System of National Accounting (SNA) standards in the 1993 United Nations Edition; this standard enumerates all the various purchases and sales in the economy.

To this end, we look at the fact that people usually either buy tangible goods and services or buy intangible goods and services, which in the latter part there are such as bonds, stocks, insurance securities, and all intangible assets that can be traded which might be named as financial or speculative transactions. In particular, speculative demand, like transactional demand, creates value-added and creates a part of society's income and output. Thus, financial operations create value-added and are considered as the sale and purchase of assets, which are included in the calculations of national accounts. Sales that generate positive or negative value-added can all be earned and entered into national accounts based on SNA calculations. Sales include market products, self-made products, and other non-market products. These operations apply to both goods and services, as well as to the transfer of assets, both tangible and intangible ones. That is, whenever a transaction is made or transferred in the economy, the value of the traded goods or assets constitutes the value of the transactions and the difference between the seller's profit and the nominal value of the goods or services or assets purchased is equal to the value-added resulting from this deal. Simply put, the sum of value-added from the components of GDP and the sum of the value of goods sold makes up the total value of transactions in the economy, and since the latter figure is equal to the value of intermediate goods plus the sum of the value-added; as a result, it also forms the aggregate supply figure. In terms of national accounting, we conclude that the aggregate nominal supply is equal to the aggregate nominal value of transactions in the economy. The above simple concept in practice involves a lot of computational complexity to reach the aggregate supply figure, but the complexity of computation does not change the basic concepts of the subject matter. So Fisher's relationship regarding this section and the previous section can be written as follows:

$$M.V = P.t = T = AS = P.as \quad (5)$$

Where "as" stands for real aggregate supply and "AS" stands for nominal aggregate supply.

Expanding the Quantity Theory of Money

Here, the income method is used to calculate the Gross National Product. In this method, the calculation of total economic activities is obtained by summing up all the different types of income. Therefore, the Gross National Product is calculated from the total money value of various incomes, including salaries, interest, and rent, which individuals earn from the production of the final goods and services.

On the other hand, liquidity is equal to:

⁹ Bidabad, Bijan (2019). Money-Transaction-Income Process: Quantification of Quantity Theory of Money. Australian Finance & Banking Review, 3(1), 33-42.

<http://www.bidabad.ir/doc/MTV4.pdf>

<https://www.cribfb.com/journal/index.php/afbr/article/view/300>

<http://www.bidabad.ir/doc/MTV4.ppt>

$$\boxed{\text{Liquidity} = \text{money} + \text{quasi-money}}$$

$$\boxed{\text{Money} = \text{banknotes and coins in the hands of the public} + \text{demand deposits}}$$

So:

$$M = CU + DD + TD \quad (6)$$

M: Liquidity

CU: Banknotes and coins in the hands of the public

DD: Demand Deposits

TD: Time Deposits

If we follow the method of calculating the total value added in the economy from the viewpoint of payment to the factors of production, we can calculate the aggregate supply through the sum of paid salaries, rents, and interests. That is:

$$M.V = P. [(Rent + Salary) + R] \quad (7)$$

R: Interest paid

Rent: Paid rents

Salary: Wages and Salaries

Therefore, by replacing the two above equations for M, we will have the following relation:

$$(CU + DD + TD).V = P. [(Rent + Salary) + R] \quad (8)$$

On the other hand, we have:

$$\boxed{\text{Interest paid} = \text{Deposit amount} \times \text{Interest rate}}$$

Therefore, it can be written as:

$$R = TD.r \quad (9)$$

Where r stands for interest rate. By placing relation (9) in (8), we will have:

$$(CU + DD + TD).V = P. [(Rent + Salary) + TD.r] \quad (10)$$

We get the above relation in terms of "r":

$$r = \frac{(CU+DD+TD).V - P.(Rent + Salary)}{TD} \quad (11)$$

This relationship shows the relationship between the reduced form of interest rates and the general level of prices, which can be shown in the following regression:

$$r_t = \beta_0 + \beta_1.P_t + u_t \quad (12)$$

By estimating the above regression, if the presented theory is true, we should reach the following results for the regression parameters:

$$\beta_0 = \frac{(CU+DD+TD).V}{TD} \quad (13)$$

$$\beta_1 = \frac{(Rent+Salary)}{TD} \quad (14)$$

Therefore, the above two parameters must be tested to see if they are true or not.

The above discussion was from the debit side of the consolidated banking system account. We extend the same subject from the assets side of the consolidated banking system account. To complete the Fisher

equation, we further expand this relationship to include transactions in the foreign sector of the economy. Here, we generalize Fisher's relation to a model that, from a monetary point of view, includes both external and internal parts. In this regard, the main viewpoint of monetarists in the "monetary approach to the balance of payments" is considered, which states the characteristics of the link between foreign and domestic currency. Therefore, in this section, we re-examine the equilibrium relationship between the money and commodity markets when the foreign sector takes the role in the market equilibrium, and to do this, we separate Fisher's equation into some of the required factors:

$$M \cdot V + M^* \cdot V^* = \sum P \cdot q \quad (15)$$

Where q is the number of goods and P is the prices of goods, and V and V^* are the velocities of domestic and foreign currencies (inside the economy).

In this equation, the left side consists of four main components: the amount of domestic money, the amount of foreign currency, the velocity of domestic money, and the velocity of foreign currency. In the above equation, each of the variables is related to each other as an aggregate in proportion to the other variables. This equation establishes the balance of external and internal sectors regarding the above variables and within the framework of the money and commodity markets' equilibrium. That is, the total value of goods sold is equal to the total amount of domestic and foreign currency exchanged in the economy.¹⁰ According to the previous explanations about the conformity of the definition of aggregate nominal supply with the "value of transactions", it can be written:

$$M \cdot V + M^* \cdot V^* = P \cdot as \quad (16)$$

Liquidity in the whole economy ($M2$) is equal to the supply of monetary resources of the banking system, including the central bank and commercial and specialized banks, and the following definitional relationship is always established in terms of banking resources and expenditures:

$$NFA + NDA \equiv M2 \equiv TD + DD + CU \quad (17)$$

NFA stands for net foreign assets of the banking system and NDA for net domestic assets of the banking system, $M2$ liquidity, TD for time deposits and DD for demand deposits in the banking system, and CU banknotes and coins in the hands of the public. Now, instead of M and M^* in the above relation, we can equate the net domestic assets and net foreign assets of the banking system. Multiply the velocity of each type of (domestic and foreign) money by the volume of the same type of money. In this case, we multiply the velocity of the domestic money by the volume of the domestic money and the velocity of the foreign money by the volume of the foreign money.

$$NDA \cdot V + NFA \cdot V^* = P \cdot as \quad (18)$$

NDA as "Net Domestic Assets" of the banking system includes net claims of the banking system from the private sector, net claims of the banking system from the public sector, and net other claims and capital accounts of the banking system. This variable is considered as the supply of financial resources, which is considered a positive function of interest rates; because with rising interest rates, banks are lending more.

NFA is the "Net Foreign Assets" of the banking system, which according to the "Monetary Approach to Balance of Payments" theory should be equal to the "Balance of Payments". The balance of payments consists of two components: the current balance and the capital balance. The first balance is a function of the exchange rate, and the second balance, according to the interest rates parities theory, is a function of the difference between domestic and foreign interest rates. So the above relation can be presented as follows, which is the extended Fisher's relation:

¹⁰ Bidabad. Bijan (2019) Controlling Parallel Exchange Market by Monetary Targeting and Anti-Inflationary Policies. Asian Finance & Banking Review, 3(1), 44-73.

<https://www.cribfb.com/journal/index.php/asfbr/article/view/295>
<http://www.bidabad.ir/doc/english-control-arz-2.pdf>

$$NDA(r).V = NFA(e,r).V^* = P . as \quad (19)$$

Foreign assets, after entering the country, are converted into domestic currency and circulated in the country's economic system. So we may assume that the velocity of foreign currency is equivalent to the effect of domestic money circulation multiplied by the exchange rate. The reason for this is the accounting operation of converting foreign currency into domestic currency. Thus:

$$V^* = e.V \quad (20)$$

This is actually done by the banking system, and if one dollar enters the country and is converted into Rials, the effect of the number of equivalent Rials to one dollar is equal to one dollar in the economy. Therefore, we can conclude that the effect of foreign currency circulation is equal to the effect of domestic money circulation multiplied by the exchange rate of conversion of foreign currency into domestic currency. This relationship expresses the banking accounting operations of converting two currencies into each other.

$$NDA(r).V + NFA(e,r).e.V = P . as \quad (21)$$

Here the income method was used to calculate the Gross National Product. In the income method, the calculation of total economic activities is obtained by adding all the different types of income. Therefore, GDP is calculated from the total money value of the various incomes, including salaries, wages, rents, and interest paid that individuals earn in the process of producing the final goods and services.

If we write the national income from the method of expenses (paid to the factors of production) and write the liquidity in terms of debts, we will have:

National income = Rent + Interest paid to the capital owners + Wages and Salaries

$$NDA(r).V + NFA(e,r).e.V = P . t = AS \quad (22)$$

$$NDA(r).V + NFA(e,r).e.V = P . [Rent + Salary + R] \quad (23)$$

So:

$$[NDA(r) + NFA(e,r).r].V = P . [Rent + Salary + R] \quad (24)$$

On the other hand, according to (9), we have Interest paid equals to the time deposit amount multiplied by the interest rate. Using (9), we will have:

$$[NDA(r) + NFA(e,r).e] = P . [Rent + Salary + TD . r] \quad (25)$$

We get the above relation in terms of P:

$$P = \frac{[NDA(r) + NFA(e,r).e]}{[(Rent+Salary)+TD.r]} . V \quad (26)$$

It is observed that in the long run, the general level of prices (P) is a function of the exchange rate (e) and the interest rate (r), and the above model should be estimated and tested. For this purpose, we take a logarithm from both sides of the above relation:

$$\ln(P) = \ln[NDA(r) + NFA(e,r).e] + \ln(V) - \ln[(Rent + Salary) + TD.r] \quad (27)$$

So we will have:

$$\ln(P) = \ln[NFA(e,r).e] + \ln \left[1 + \frac{NDA(r)}{NFA(e,r).e} \right] + \ln(V) - \ln(TD.r) - \ln \left[1 + \frac{Rent+Salary}{TD.r} \right] \quad (28)$$

$$\ln(P) = \ln(NFA(e,r)) + \ln(e) + \ln \left[1 + \frac{NDA(r)}{NFA(e,r).e} \right] + \ln(V) - \ln(TD) - \ln(r) - \ln \left[1 + \frac{Rent+Salary}{TD.r} \right] \quad (29)$$

And in the regression form:

$$\ln(P) = \beta_0 + \beta_1 \ln(r) + \beta_2 \ln(e) + \beta_3 \ln(\text{NFA}) + \beta_4 \ln\left(1 + \frac{\text{NDA}}{\text{NFA.e}}\right) + \beta_5 \ln\left(1 + \frac{\text{Rent+Salary}}{\text{TD.r}}\right) + \beta_6 \ln(V) + \beta_7 \ln(\text{TD}) + u_t \quad (30)$$

If we replace $X = 1 + \frac{\text{NDA}}{\text{NFA.e}}$ and $Y = 1 + \frac{\text{Rent+Salary}}{\text{TD.r}}$ in the above regression equation, then we have:

$$\ln(P) = \beta_0 + \beta_1 \ln(r) + \beta_2 \ln(e) + \beta_3 \ln(\text{NFA}) + \beta_4 \ln(X) + \beta_5 \ln(Y) + \beta_6 \ln(V) + \beta_7 \ln(\text{TD}) + u_t \quad (31)$$

The above equation is a regression in which u_t is defined as the error term. The relationship between interest rates and the general price level is expected to be asymmetric for the following reasons:

1- Share of financial costs in the markup prices of goods and services: Financial costs are added to the markup prices of goods and services as a part of the costs of the firms. Therefore, when the interest rate of bank loans increases, then the financial cost of the firm will increase too. As a result, firms add the financial costs to their markup costs as a part of their costs, and the firm supplies its products at higher prices. Since these prices are included in the consumer price index with their specific weight, higher prices lead to an increase in the general level of prices. On the other hand, with lower interest rates on loans, although the financial cost of the firm is reduced, firms generally do not reduce the price of their manufactured goods in the same proportion. Therefore, it can be seen that the increase in the interest rates on loans has increased the general level of prices, but the decrease in interest rates on loans does not reduce the general level of prices in the same proportion, and there is an asymmetric relationship between interest rates and general level of prices.

2. Interest rates and the general price level will affect the economy through different time delays. For example, if the money supply increases, it will increase the general price level and decrease interest rates. The important point is that the period of increase in the general level of prices after the increase in money supply is different from the period of decrease in interest rates after the money supply increase, and this will cause an asymmetric relationship between interest rate and price.

3. Duration of loan agreements: If the interest rate of the loan increases or decreases, it does not affect the previous agreements, and we have to wait for a while until the interest rate of the loan enters the banking transaction cycle and affects the general level of prices. Therefore, this issue can also cause an asymmetric relationship between interest rate and price.

4. Fixed interest rates on long-term investment deposits: As interest rates on deposits increase and decrease, depositors with long-term investment deposits should wait for their contracts to expire.

Previous Studies

Irving Fisher (1867-1947) was the pioneer in addressing money, inflation, and interest rates. Fisher's ideas are the base for the development of the purchasing power parity concept. The nominal interest rate i_t is assumed to consist of two components: the expected inflation rate (π_t^e) and the real interest rate r_t and the subscript t stands for time.

$$i_t = r_t + \pi_t^e \quad (32)$$

He claimed that there is a one-to-one relationship between interest rates and expected inflation and that the real interest rate is independent of inflation. Accordingly, Fisher's equation can be written as follows:

$$(i_t + 1) = (r_t + 1)(E_{t-1}\pi_t + 1) \quad (33)$$

π_t denotes inflation and E_{t-1} stands for expectations at the time $t-1$. By simplifying the above equation, we will get the following equation:

$$i_t = r_t + E_{t-1} \pi_t + r_t E_{t-1} \pi_t \quad (34)$$

Fisher assumes that the expression to the right of the above equation is too small and therefore omits it. So we will have:

$$i_t = r_t + E_{t-1} \pi_t \quad (35)$$

Which leads to (32). And this equation formulates the basis of Fisher's hypothesis that the nominal interest rate is equal to the sum of the real interest rate and expected inflation. In the advanced case, Fisher's hypothesis states that the real interest rate is constant over time and is determined only by real factors. The Stochastic form of Fisher's hypothesis is:

$$i_t = \beta_0 + \beta_1 \pi_t^e + u_t \quad (36)$$

In his analysis, Fisher used inflation data and interest rates of Britain and the US for the periods of 1820-1924 and 1890-1927. His findings show that there was "no obvious correlation" between the short-term price changes and interest rates in these countries, and the correlation coefficients were -0.459 for UK and -0.289 for the US without the presence of lag in the data. In contrast, when the previous-period inflation (inflation with a lag period) was used as the expected proxy inflation, the correlation coefficients increased significantly. When price changes spread over 28 years and 20 years, the correlation coefficients were 0.98 and 0.875 for Britain and the United States, respectively.

Based on Fisher's results, it can be said that the nominal interest rate should follow the expected inflation. In other words, the causality should be from the expected inflation to the nominal interest rate.

Sam Taban, Teyfur Bayat, and Ferit Ounder (2014) studied the nominal interest rate and the consumer price index relationship in Austria using quarterly data from 1990 to 2013 to test Fisher's hypothesis. The findings of their study show that according to Granger's linear causality test, there is no causal relationship between interest rate variables and inflation in Austria. The results of the frequency domain causality test indicate that the two-way causality relationship is valid in the short run, but in the long run, the causality relationship is from inflation to interest rate.

Stephen Piccinino (2011) examined Fisher's hypothesis for the euro area from 1999 to 2011. In this study, The "European Interbank Offered Rate" for the interest rate variable and "German Federal Securities" with six months of maturity were used to measure expected inflation. The results showed that Fisher's hypothesis holds when the whole data set is considered. However, the two variables relationship was not established for the period from September 2008 to March 2011, and this was probably due to the tight monetary policy implemented by the authorities.

Ayub G et al. (2014) examined the inflation and interest rate relationship in Pakistan for the period 1973 to 2010, using Fisher's hypothesis to show the long-run equilibrium relationship between the two variables. Johansson and Granger's cointegration technique (based on residual) was used to experimentally investigate the nominal interest rate and inflation rate relationship. The results indicated that there is a long-term equilibrium relationship between nominal interest rates and inflation rates for the period 1973-2010 in Pakistan.

Lardic and Mignon (2003), in their study using Engel-Granger cointegration, investigated the interest rates and inflation rate relationship in G7 countries. The results of their studies showed that there is a long-term positive relationship between interest rate and inflation rate.

Milion (2003) examined the long-run relationship between nominal interest rate and inflation using US data. The results showed that the US Federal Reserve lowered the nominal interest rate when inflation rose and increased it when inflation fell. It is argued that the Fed policymakers have often pursued a policy of price stability by adjusting their operations to inflation. Overall, the results of this study confirmed Fisher's basic theory of interest rate and inflation rate.

Booth and Ciner (2001) applied the integration technique to examine the interest rates and inflation relationship in nine European countries and the United States and concluded that, with just one exception, there is a long-term relationship between inflation and interest rates.

Azwifaneli (2016) investigated the nominal interest rate and inflation rate relationship by using quarterly

data for the period of 2001-2014 in South Africa. The results showed that in the long run, nominal interest rate and inflation move together, but there is no one-to-one relationship between them. Therefore, a change in the nominal interest rate will cause a smaller change in the inflation rate, and hence the real interest rate will increase. Therefore, Fisher's relationship was not established fully in South Africa.

Navoda Edridighe, Selliah Sivarajasingham, and John Nigel (2015) examined the relationship between inflation and interest rates in Sri Lanka for the period 1959 to 2011. Their main goal was to study the short-term and long-term effects of the Fisher hypothesis in that country. The results of the study indicated that in the long run, the nominal interest rate is fully adjusted by expected inflation. In the short run, there is a significant positive relationship between nominal interest rates and expected inflation, but the Fisher's effect was not fully observed. Therefore, changes in the monetary policy instruments to achieve low inflation did not seem to be fully effective.

Hossein Asgharpour, Nader Mehregan, and Morteza Ezzati (2006) studied the causal relationships between inflation and interest rates based on inter-states observations (panel data from 24 countries) from 2001 to 2003. Most causal tests performed were mainly the Granger causality test and the Hsiao test. The results showed that an increase in interest rate had caused an increase in inflation, and thus the interest rate is the cause of inflation, but a significant increase in inflation has not been able to increase the interest rate in the sample countries. The results showed a one-way causality from interest rate to inflation.

Parviz Davoodi and Mehdi Zolghadri (2011) studied the interest rates and inflation rates relationship in the Iranian economy for the period 1960 to 2010. In this study, composite tests in three different scenarios were used to examine the interest rate and inflation rate relationship. The results show that there is a long-term equilibrium relationship between the interest rate and the inflation rate. By using the Granger-Sims causality test, based on the error correction model, the causality relationship between the two variables was examined in the short run, and the causality relationship from interest rate to inflation rate was confirmed. The causal relationship from inflation to interest rate was also examined, and for each of the three scenarios, the result showed that the coefficient of variation for the inflation rate is not significant, and inflation rate cannot be the cause of interest rate, and therefore the causal relationship between interest rates and inflation rates is one-way. The results showed that the increase in interest rates had caused the increase in inflation, and thus the interest rate is the cause of inflation, but the increase in inflation has not been able to significantly increase interest rates in Iran. Therefore, the results of studies show a one-way causality from interest rate to inflation.

Doman Bahrami Rad and Akbar Komijani (2008) examined the long-run relationship between bank loan interest rates and inflation rate in Iran with the Johansson Co-integration test and the Granger causality test for the period of 1973-2005. The variables of the weighted interest rate of bank saving deposits, the weighted interest rate of bank loans, the total weighted interest rate of bank deposits and inflation rate, and the rental rate of return on housing in urban areas were used as alternatives to the nominal interest rate. The results of this study indicated that there is no causal relationship between interest rate variables and the inflation rate in the short run and a causal relationship from the inflation rate to the nominal interest rate in the long run. Therefore, in the Iranian economy, changes in nominal interest rates, in the long run, can be explained by changes in inflation.

Hassan Khodavisi and Ali Khajeh Mohammadlou (2016) studied the relationship among exchange rate, inflation rate, and interest rate under the viewpoint of Fisher's theory for the Iranian economy using the Vector Autoregression (VAR) method from the period 1981 to 2004. The results of the Johansen test and the estimation of the Vector Error Correction Model (VECM) showed that in the long run, the inflation rate has a significant negative effect, and the exchange rate has no effect on the interest rate. The results of the Johansen test and estimation of the VECM showed that there is a long-run equilibrium relationship between the variables in the model so that interest rate elasticity for the exchange rate is not significant and exchange rate increase (decrease) does not affect the interest rate. Also, the elasticity of interest rates

to inflation was equal to -1.16; that means an increase in inflation has a significant negative effect on the interest rate. The study of the short-term relationships showed that in the short run, interest rate elasticity to the exchange rate was positive and significant, but interest rate elasticity to the inflation rate was not significant and had no effect on interest rates. In other words, Fisher's theory of the international effect on the Iranian economy was rejected.

Abbas Ali Abu Nouri, Teymour Mohammadi, and Somayeh Sadat Sajjadi (2012) investigated the relationship between inflation and bank deposits interest rates in the Iranian banking system using cointegration error correction models. Their study showed that there is an inverse relationship between inflation and deposits' interest rates.

Mohammad Taher Ahmadi Shadmehri, Mohammad Ali Fallahi, and Somayeh Khosravi (2011) examined the Hsiao causal test for interest rate and inflation for the MENA countries from 1997 to 2008. The results showed that only in the case of Djibouti there is a causal relationship from interest rates to inflation. But in other countries, the change in interest rate was not the cause of the change in inflation.

Empirical Investigation

Threshold autoregressive models can record asymmetric and nonlinear movements of variables. In nonlinear models, the reaction of one variable to changes in other variables is examined nonlinearly. In this regard, the threshold autoregressive models can be classified as nonlinear models.

A crucial statistical issue is the test of linearity versus nonlinearity. Linearity is a basic premise among many economists who use linear models unless there is convincing evidence to prove that it is nonlinear. Threshold models can be tested for linearity or nonlinearity. Most previous experimental studies have used linear models in time series, which is based on the assumption that the mode of adjustment toward long-run equilibrium is necessarily symmetric. However, the assumption of symmetric adjustment does not always exist, and it is often argued that some fundamental economic variables are adjusted asymmetrically; therefore, these variables cannot be modelled by linear models. In this paper, the Fisher model is investigated in a nonlinear framework.

Smooth Transition Autoregressive Model (STAR)

Smooth Transition Autoregressive models are one of the types of nonlinear time series regression models that can be considered as an extended form of the Switching Regression Models introduced by D.W. Bacon and D.G. Watts (1971). The researchers considered two regression lines and designed a model in which the transition from one line to another occurs smoothly. However, in the time series literature, Granger, C.W., and T. Terasvirta (1993) first described the STAR model in their studies. It is worth mentioning that the mentioned model can be used in two forms: Exponential Smooth Transition Autoregressive (ESTAR) and Logistic Smooth Transition Autoregressive (LSTAR) as follows:

$$Y_t = \alpha + \varphi'z_t + \theta'z_t F(s_t) + u_t = \alpha + \{\varphi + \theta F(s_t)\}'z_t + u_t \quad t = 1, \dots, T \quad (37)$$

$$F(s_t) = \frac{1}{1 + \exp[-\gamma(s_t - c)]} \quad \text{for LSTAR function} \quad (38)$$

$$F(s_t) = 1 - \frac{1}{\exp[-\gamma(s_t - c)^2]} \quad \text{for ESTAR function} \quad (39)$$

Where Y_t is the dependent variable, α is the intercept, and z_t is the vector of the explanatory variables. In the above regression, the coefficients of the explanatory variables are not constant and are a function of the s_t variable. $F(s_t)$ is called the transfer function, and s_t is the transition variable, c is the locational parameter, and $\gamma > 0$ is called the parameter or slope of transition. s_t can be any of the model's variables of z_t , their lags, or a variable outside the model. The above specification shows that the model can also be interpreted as a linear function with coefficients that change over time randomly.

For the LSTAR model, the coefficients of $F(s_t)$ as a function of s_t change uniformly from φ to $\varphi + \theta$ when s_t moves from $-\infty$ to $+\infty$. But for the ESTAR function, when s_t moves from c to $\pm\infty$, the coefficients change symmetrically around the "c" midpoint from φ to $\varphi + \theta$. Therefore, LSTAR can model the symmetric behaviour of variables. For example, this model is an appropriate model for describing the processes that behave differently during periods of prosperity than periods of recession, and the transition from one regime to another is done smoothly. On the other hand, the ESTAR model is suitable for situations where the coefficients or dynamic adjustment process behave similarly at the upper and lower limits levels of s_t and show different behaviour only at mid-values. When the slope parameter $\gamma = 0$, the transition function will be $F(s_t) = 1$, so the STAR model becomes a linear model. On the other hand, when $\gamma \rightarrow \infty$ the LSTAR model becomes a regression model of the state change with two discrete regimes. In the ESTAR model, if $\gamma \rightarrow \infty$, we actually get into a linear model.

Testing against Nonlinearity

Before specifying and estimating a STAR nonlinear model, the model must be first tested against the nonlinearity. If the null hypothesis that the model is linear is rejected, the potential nonlinear (LSTAR or ELSTAR) model should then be selected among the potential nonlinear models to be estimated. To test the linearity hypothesis, the $\gamma = 0$ limitation in the nonlinear model (37) should be tested. The problem here is that the model coefficients will not be detectable under the null hypothesis $H_0: \gamma = 0$. Therefore, for this test, we use the transfer function approximation based on the Taylor expansion as follows:

$$Y_t = c + \beta'_0 z_t + \sum_{j=1}^3 \beta'_j z_t s_t^j + u_t^* , \quad t = 1, \dots, T \quad (40)$$

The linear model is tested based on the null hypothesis $H_0: \beta_1 = \beta_2 = \beta_3 = 0$ based on Lagrange coefficient statistics or F ratio. There are two ways to select a transition variable (that is, the variable under which the model parameters change). The first method is to use theory. Of course, when the theory itself is to be tested, or we try to test competing theories, this method can no longer be acceptable and used. The second method for selecting the transition variable is to use statistical tests. According to Terasvirta (1998), after estimating the model with different transition variables, any variable that rejects the assumption of zero linearity is selected as the transition variable. If several variables reject the null hypothesis, a variable should be used among the potential transition variables to estimate the nonlinear model that minimizes the p-value of the test.

After the rejection of the linearity assumption and selection of the transition variable, the next step is to select the nonlinear model type. In the STAR models, there is no explicit economic theory for choosing the type of model. Therefore, the choice of STAR model type (between ESTAR and LSTAR) should be based on data and statistical tests. For this purpose, we perform the following tests based on Equation (40):

$$H_{02}: \beta_1 = 0, \beta_2 = \beta_3 = 0$$

$$H_{03}: \beta_2 = 0, \beta_3 = 0$$

$$H_{04}: \beta_3 = 0$$

If H_{03} were rejected, and the other two hypotheses were accepted, and the ESTAR model was selected. If H_{04} or H_{02} were rejected; the model is LSTAR. In addition, if all three null hypotheses were rejected, we consider the strongest rejection of the null hypothesis, given the p-value. According to the proposed rule, if the H_{03} hypothesis is rejected most strongly; the model is selected as ESATR and otherwise LSTAR.

It should be noted that most research in the field has used linear models. Since the linear model is often not able to express the gradual changes of variables in different economic situations, modelling the nonlinear economic variables has been considered by many economists. Of course, it should be noted that in many models, linear regressions provide a better response than nonlinear regression; in other words, there are necessary conditions that require linear or nonlinear model estimation. Up to the point that the

necessary conditions have not been proved, the researcher cannot use these models solely because nonlinear models better show gradual changes. In this regard, after examining the nonlinear conditions, the Smooth Transition Autoregressive model has been used to investigate the inflation and interest rates relationship in Iran.

Variables Stationarity

The results of the generalized Dickey-Fuller test and the Phillips-Prone test are shown in Table (1), accompanying the test statistics with the critical values. Accordingly, the LNP and LNX variables are stationary at the level (with and without trend). And other variables at the level (with the trend and without the trend) are non-stationary and are stationary with their first-order difference. The existence of a nonlinear relationship among variables was tested. If the existence of a nonlinear relationship is confirmed among the variables used in the model, the appropriate transfer variable and the number of nonlinear model regimes should be determined based on F, F2, F3, and F4 test statistics. The estimation results of this stage are presented in Table (2). Given the probabilistic value of the F-statistic in Table (2), the null hypothesis of this test is based on the linearity of the model for the variables LNNFA, LNr, LNTD, LNV, LNNFA, and LNe is rejected, and the assumption of nonlinear relationship for these variables is accepted. The next step is to select the appropriate transfer variable from the possible transfer variables for the nonlinear model. Any potential variable can be considered to be selected as the transfer variable, but the priority is with the transfer variable whose F test of the null hypothesis is more strongly rejected. Based on this, the most appropriate transfer variable is determined according to Table (2), and the research hypotheses, the LNr variable, is selected. Selecting the appropriate model for the transfer variable according to the statistics of the F2, F3, and F4 tests is the next step in estimating the STAR model. According to Table (2), the proposed suitable model for the LNr transfer variable ESTAR model, i.e., the exponential smooth transition model with a threshold point, is selected.

Table (1): Tests to determine the stationarity of variables

Variable	Augmented Dickey-Fuller unit root test				Phillips-Peron unit root test	
	Without trend	Critical value 5%	With trend	Critical value 5%	Without trend	With trend
LNP	4.571	-2.292	-4.286	-3.518	-4.353	-4.215
LNr	-1.138	-2.292	-1.863	-3.518	-1.127	2.021
LNe	-1.354	-2.931	-2.095	-3.518	-1.086	-1.489
LNX	-4.115	-2.292	-5.934	-3.518	-4.144	-5.921
LNY	-1.630	-2.294	-1.745	-3.536	-4.649	-2.942
LNnfa	-0.140	-2.292	-2.203	-3.518	-0.246	-1.765
LNV	-1.255	-2.292	-1.348	-3.515	-1.619	-1.750
LNTD	0.833	-2.292	-0.812	-3.515	0.611	-1.201
dLNP	-7.759	-2.931	-7.688	-3.518	-14.829	-17.462
dLNr	-5.506	-2.931	-5.452	-3.518	-5.506	-5.452
dLNe	-4.127	-2.931	-4.214	-3.518	-4.004	-3.979
dLNX	-15.709	-2.931	-7.543	-3.520	-18.100	-18.984
dLNY	-14.338	-2.943	-13.277	-3.536	-5.452	-6.819
dLNnfa	-5.824	-2.931	-5.879	-3.518	-5.836	-5.881
dLNV	-5.474	-2.931	-5.412	-3.518	-5.666	-5.615
dLNTD	-5.203	-2.931	-5.397	-3.518	-5.241	-5.366

Source: Research Findings

Table No. (2): Model type and transfer variable

Suggested model	Statistical probability value F2	Statistical probability value F3	Statistical probability value F4	Statistical probability value F	Transfer variable
Linear	0.0152	-	-	-	LNY(t)
ESTAR	0.9145	0.0060	0.1991	0.0295	LNe(t)
ESTAR	0.2926	0.0076	0.0499	0.0013	LNNFA(t)
ESTAR	0.3079	0.0211	0.1583	0.0285	LNr(t)
ESTAR	0.2300	0.0011	0.2159	0.0057	LNTD(t)
LSTAR	0.0313	0.0971	0.0257	0.0036	LNV(t)
Linear	0.0971	-	-	-	LNx(t)

Source: Research Findings

Due to the nonlinear nature of these models, this estimation step begins with finding the appropriate initial values for estimating the model. Using these initial values, the Newton-Raphson algorithm was applied, and by maximizing the ML function, the parameters are estimated. The results have been reported in Table (3).

Table No. (3): Model estimation results

	Estimate	t-stat	p-value
Linear Part			
Const	-3.50	-1.46	0.157
LNY(t)	29.02	1.57	0.128
LNe(t)	0.24	0.54	0.591
LNNFA(t)	-0.61	-2.25	0.033
LNr(t)	0.25	0.24	0.808
LNTD(t)	0.76	1.68	0.105
LNV(t)	0.99	1.07	0.293
LNx(t)	409.70	1.89	0.071
Nonlinear part			
Const	5.21	1.78	0.087
LNY(t)	-78.47	-0.99	0.331
LNe(t)	0.58	0.68	0.504
LNNFA(t)	1.01	3.11	0.005
LNr(t)	1.53	1.14	0.264
LNTD(t)	-1.93	-2.78	0.010
LNV(t)	0.90	2.32	0.029
LNx(t)	-198.46	-0.63	0.535
Adjusted R ² : 0.667 AIC: -1.781 SC: -1.0029 HQ: -1.494			

Source: Research Findings

The estimated final values for the uniformity parameter (γ) is 368.7, and the logarithmic threshold value of the interest rate (c) is 2.36. Therefore, the transfer function will be as follows:

$$(Y, C, S_t) = F(s_t) = 1 - \frac{1}{\exp[-\gamma(s_{T-} - c)^2]} = G(368, 2.3, L, r(t)) = 1 - \frac{1}{\exp[-36(LNr(t) - 2.3)^2]} \quad (41)$$

In the first regime, it is $G = 0$, and in the second regime, it is $G = 1$. So for the first regime, we have:

$$LNP(t) = -3.5 + 29 L Y(t) + 0.8 L e(t) - 0.6 L NFA(t) + 0.2 L r(t) + 0.7 L TD(t) + 0.9 L V(t) + 409.7 LNx(t) \quad (42)$$

And for the second regime, we will have:

$$\text{LNP}(t) = 1.7 - 49.4 \text{ LNY}(t) + 0.8 \text{ LNe}(t) + 0.4 \text{ LNNFA}(t) + 1.7 \text{ LNr}(t) - 1.1 \text{ LNTD}(t) + 1.8 \text{ LNV}(t) + 211.2 \text{ LNX}(t) \quad (43)$$

Therefore, it can be inferred that the interest rate has affected the inflation rate nonlinearly and asymmetrically during the study period, so that in the first regime (where the logarithm of the interest rate is less than 2.36 (interest rate less than 10.6%)) The effect of interest rates on inflation is less than the second regime (interest rates above 10.6%). Therefore, it can be concluded that by increasing interest rates (particularly when the interest rate exceeds the threshold), i.e., 10.6 percent, its impact on inflation increases.

A noteworthy point is the effect of the money velocity on inflation at the same time as interest rates increase, and as it can be seen at high interest rates, the inflationary effect of the velocity of money increases. Also, the effect of the exchange rate on the inflation rate is positive, and with the increase in the interest rate, the influence of the exchange rate on the inflation rate also increases.

According to the logistic function related to the regime change, the moment of regime change can be considered for the estimated model. At $\text{LN}r = 2.61$, the value of the transfer function is $G(Y, c, S(t)) = 0.5$, and considering that the uniformity parameter (Y) is estimated to be 10, the transfer between two limit regimes $G(Y, c, S(t)) = 0$ and $G(Y, c, S(t)) = 1$ is done slowly. Therefore, along with the basic assumption of the Smooth Transition Autoregressive model, it can be said that the process of regime change in the effect of interest rates on inflation has not drastic changes around the threshold point, and changes in parameters have been slow

Conclusion

In this study, the effect of interest rate on the general price level was investigated. For this purpose, the nonlinear regression model of Smooth Transition Autoregressive was used for the period 1973 to 2017. The results indicate that interest rates asymmetrically affect the general level of prices. Therefore, it can be concluded that with the increase of interest rates (particularly when the interest rate exceeds the threshold), its impact on the inflation rate increases. The effect of the money velocity on inflation at the same time increases as interest rates increase, and at high-interest rates, the effect of the velocity of money on inflation increases faster. Also, the effect of interest rates on the general price level has no severe effect, and changes in the parameters were slow. Therefore, it is concluded that interest rate as a policy instrument is not effective for controlling high inflation. Moreover, using those policies that diminish the velocity of money circulation is more effective to control prices.

Generally, the result shows that in the long run, interest rate yield by the economy due to the payment to the production factors practically vanishes and is eliminated by lowering the purchasing power of money due to the price increase in the economy. This might be considered as one of the wisdoms that the Almighty God has said in the Holy Qur'an about the prohibition of usury that: "**God effaces usury**"¹¹ means that Almighty God eliminates the effects of the usury¹² income increment in the economy.

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