# Mesoeconomics of Migration and Trade 

Bijan Bidabad ${ }^{1}$

Mahshid Sherafati ${ }^{2}$


#### Abstract

In this paper, we analyze Mesoeconomics of migration and trade in interregional and international economy. Decision-makers are interested in analyzing how changing the prices or wages will lead to these movements and what policies can be devised to revert the markets into their initial equilibrium.


We assume in our simple model, that the wages or prices change in one region, and we will measure the equilibrium of the markets of the two regions. After some time, by migration of workers and trading commodities, a new equilibrium will be achieved. If one of the governments of the two regions intends to revert the equilibrium to the state before changes were applied, how should that government act and how great will its financial burden be? In this paper, we try to formulate the answer to this question in the context of two simple, homogenous, and similar-shape economies.

This paper is centered on the behavior of individuals of the two regions, which we try to generalize it so that we could evaluate the problem at meso level analytically. Therefore, we simplify the problem to the extent that its micro and macro dimensions coincide.

We check different cases of Changing wage elasticity of price or production elasticity of employment and examine the financial burden of policy of no labor and commodity movement. In this way, if the government of region 1 decides to counteract and revert the conditions into the previous state it can pay specific subsidies to workers, legislate due taxes on selling commodities, and thereby establish the conditions of equation as aprior to changes.

Keywords: Mesoeconomics, Trade, Migration, Regional economic policy

## Introduction

Labor and commodity movement is of high importance in interregional and international. Decisionmakers are interested in analyzing how changing the prices or wages will lead to these movements and what policies can be devised to revert the markets into their initial equilibrium. The financial burden of such policies is the main factor for employing them. Consider that there is a general equilibrium in all the markets of two regions, between which labor movement and commodity trade occur easily. If we assume that in a special situation and in an exogenous manner, the wages or commodities' prices change in one region, the equilibrium of the markets of the two regions will be changed, and after some time, by

[^0]migration of workers and trading commodities, a new equilibrium will be achieved. If one of the governments of the two regions intends to revert the equilibrium to the state before changes were applied, how should that government act and how great will its financial burden be? The purpose of this paper is to find an answer to this question in the context of two simple, homogenous, and similar-shape economies. In other words, we attempt to analyze the exogenous changes of prices and wages in one region and estimate the financial burden for confronting these changes. The analytical dimension of this paper is centered on the behavior of individuals of the two regions, which we try to generalize it so that we could evaluate the problem macro-analytically. In order to be able to consider micro and macro perspectives simultaneously, we should simplify the problem to the extent that its micro and macro dimensions coincide. To do so, we consider two regions with two identical economies and by incorporating many strong assumptions. After finding the relationships between supply and demand for labor and commodities in both regions, it will be attempted to realize how we can stop the trade of commodities and labor forces migration by applying some implicit policies.

## Assumptions of the model

We consider two similar regions, which have identical and homogenous economies, and the following conditions apply in them:

1. In both regions, one type of commodity is produced and consumed.
2. Transportation costs between the two regions are trivial and negligible.
3. Production is done based on the labor factor, and the production function depends only on the employed labor.
4. Perfect competition is dominant in all markets of the two regions.
5. Income of people of each region is consumed in that region and no transfer payment is taken place between the two regions.
6. There exist no difficulty in changing prices and wages and they can be changed easily.
7. The considered commodity is normal.
8. There is no money illusion for the workers.
9. All the income is consumed and the amount of investments and savings is zero.
10. Exchanges are barters, and they are done in return for doing jobs.
11. The commodity quality and labor force efficiency are equal in both regions.

## Model

In order to achieve the supply and demand relations in the labor and commodity markets, we present the problem in its simplest form as follows: the supply and demand of labor and commodity are obtained from the optimal behavior of consumers and producers.

## Labor supply

Workers try to maximize their utility, which is the function of leisure and income. Their utility function is defined as follows:
$u=u(L, y)$
Where $L$ and $y$ denote leisure and income respectively. We consider the following stages for maximizing the utility. Leisure time is considered as differential of total available time for which the utility function is defined. It is written as follows:
$L=T-w$

Where $T$ signifies the total available time and $w$ indicates the working time. If we define $r$ as wage rate, the income constraint will be as follows:
$y=r w$
By replacing equations (2) and (3) in equation (1), we have:
$u=u(T-w, r w)$
Maximum of utility function (4) will determine the working time and leisure time of labor. To do so, we set the derivative of $u$ with respect to $w$ equal to zero:
$\frac{d u}{d w}=-\frac{d u}{d L}+\frac{d u}{d y} r=0$
Or, it can be written as the following:
$r=\frac{\mathrm{du} / \mathrm{dL}}{\mathrm{du} / \mathrm{dy}}$
Equation (6) is the inverse supply function for labor, in which the ratio of marginal utility of leisure to marginal utility of income is equal to the wage rate. If we inverse equation (6), we will have:
$w^{s}=w^{s}(r) \quad \frac{d w^{s}}{d r}>0$
The above equation is the labor supply equation, where the labor supply increases as the wage rate increases.

## Commodity demand

As mentioned above, demand for commodity is obtained by maximizing consumer's utility. The consumer utility function is defined as below:
$v=v(q)$
The consumer's budget constraint is:
$y=p q$
Where $p$ and $q$ are price and quantity of the intended commodity respectively. Consumer utility maximization of (8), with the constraint (9), leads to the following demand function:
$q^{d}=y / p \quad \frac{d q^{d}}{d p}<0$

## Labor demand

Producer maximizes its profit ( $\Pi$ ) according to the production function of $\mathrm{q}=\mathrm{q}(\mathrm{w})$ and the cost of rw.
$\Pi=p q(w)-r w$
Function (11) is maximized when the marginal productivity of labor equals its real wage.
$\frac{d q(w)}{d w}=\frac{r}{p}$
Therefore, demand function for labor can be written as follows:
$w^{d}=w^{d}\left(\frac{r}{p}\right) \quad \frac{\partial w^{d}}{\partial r}<0 \quad$ and $\quad \frac{\partial w^{d}}{\partial p}>0$
Since $\mathrm{w}^{\mathrm{d}}$ has an inverse relationship with $p$, for simplification, we write $\left(\frac{r}{p}\right)$ as $\left(\frac{p}{r}\right)$, and hereafter, we use the following equation instead of the above one:
$w^{d}=w^{d}\left(\frac{p}{r}\right) \quad \frac{\partial w^{d}}{\partial r}<0 \quad$ and $\quad \frac{\partial w^{d}}{\partial p}>0$

## Commodity supply

If we solve the cost equation $c=r w$ for the inverse production function $w=q^{-1}(q)$, the cost function will be obtained as follows:
$c=r q^{-1}(q)$
Commodity supply function will be attained by maximizing the profit function, which is defined as below:
$\Pi=p q-r q^{-1}(q)$
If we set the derivative of $\Pi$ with respect to $q$ equal to zero, we will have:
$\mathrm{P}=\mathrm{r} \frac{\mathrm{d} q^{-1}(q)}{\mathrm{dq}}=\frac{d c}{d q}$
Which is resulted from the equality of price and marginal cost. In other words, we will have:
$\frac{\mathrm{d} q^{-1}(q)}{d q}=\frac{p}{r}$
Provided that the price is higher than the average variable cost, then the commodity supply function will be as follows:

$$
\begin{equation*}
q^{s}=q^{s}\left(\frac{p}{r}\right) \quad \frac{\partial q^{s}}{\partial q}>0 \quad, \frac{\partial q^{s}}{\partial r}<0 \tag{19}
\end{equation*}
$$

## General equilibrium

We consider the market's supply and demand for labor and commodity as the total supply and demand for individual's labor and commodity. The general equilibrium in the market is defined as follows, which necessitates the equality of supply and demand for both:

$$
\begin{array}{ll}
w^{s}=w^{s}(r) & \frac{d w^{s}}{d r}>0 \\
w^{d}=w^{d}\left(\frac{p}{r}\right) & \frac{\partial w^{d}}{\partial r}<0, \frac{\partial w^{d}}{\partial p}>0 \\
w^{d}=w^{s} & \\
q^{s}=q^{s}\left(\frac{p}{r}\right) & \frac{\partial q^{s}}{\partial p}>0, \frac{\partial q^{s}}{\partial r}<0 \\
q^{d}=y / p & \frac{d q^{d}}{d p}<0 \\
q^{d}=q^{s} & \\
c=r w=y=p q & \tag{26}
\end{array}
$$

Equation (26) indicates national income accounting equation in our defined economy. In other words, total payment to the labor factor (rw) is equal to firm's cost (c) and is equivalent to income (y) or the production value added (pq). In equations (21) and (23), labor demand and commodity supply are functions of the ratio of commodity price to wage. If we define the term "real price" as the inverse of real wage, according to equations (13), (14), and (19), the labor demand and commodity supply will be functions of the real price. In other words, if $p$ and $r$ are increased or decreased at a same ratio, the "real price" will not change, and the labor demand and commodity supply will remain constant. Also, equations (22) and (25) will create a further equilibrium in the same rate of labor supply and commodity demand, and ratio of employment to production will again be equal to "real price" or $\frac{p}{r}$. In other words:
$\frac{w}{q}=\frac{p}{r}$
As said, this condition applies when $\mathrm{d}\left(\frac{p}{r}\right)=0$. It means that:
$\frac{d p}{d r}=\frac{p}{r}=$ Const.
If we suppose the ratio of changes of price and wage as $\eta$ times of the "real price", the wage elasticity of the price can be defined as follows using equation (28):
$\frac{d p}{d r}=\eta \frac{p}{r}$
Equation (29) defines relative changes of price, resulting from the changes of wage rate, as a multiple of "real price". If $\eta=1$, then equation (29) will be the same as equation (28), and clearly $\mathrm{d}\left(\frac{p}{r}\right)=0$ and price and wage have changed at a same rate. According to equation (29), we have:
$\eta=\frac{d p / p}{d r / r}$
Where $\eta$ is wage elasticity of price. In equilibrium, according to equation (27), it is evident that the ratio of employment to production is equivalent to real price. Therefore, by using a method like the above one, we can obtain the production elasticity of employment in this way. When $\mathrm{d}\left(\frac{w}{q}\right)$ equals zero, the relative changes of $\frac{w}{q}$ is equal to $\frac{w}{q}$ itself. I.e., according to $\mathrm{d}\left(\frac{w}{q}\right)=0$, we will have:
$\frac{d w}{d q}=\frac{w}{q}=$ Const.

Using equations (27) to (31), in equilibrium, we will have the following:
$\eta=\frac{d w / w}{d q / q}=\frac{d p / p}{d r / r}$

In other words, in equilibrium, production elasticity of employment equals wage elasticity of price.

## Generalization to two regions

The above model, which indicated the general equilibrium in the markets of an economy, can be generalized to two regions, as will be explained. We consider two similar regions with the structure of equations (20) to (26). The subscripts $i=1$, 2 imply region 1 and 2, respectively. Region 1 can be defined as a specific country and region 2 as the outside world. In other words, for each distinct region, we have:
$w_{i}^{s}=w_{i}^{s}\left(r_{i}\right) \quad \frac{d w_{i}^{s}}{d r_{i d}}>0$
$w_{i}{ }^{d}=w_{i}{ }^{d}\left(\frac{P_{i}}{r_{i}}\right) \quad \frac{\partial w_{i}}{\partial p_{i}}<0 \quad, \quad \frac{\partial w_{i}^{d}}{\partial r_{i}}<0$
$w_{i}^{d}=w_{i}^{s}$
$q_{i}^{s}=q^{s}\left(\frac{p_{i}}{r_{i}}\right) \quad \frac{\partial q_{i}^{s}}{\partial p_{i}}>0 \quad, \quad \frac{\partial q_{i}^{s}}{\partial r_{i}}<0$
$q_{i}{ }^{d}=y_{i} / p_{i} \quad \frac{d q_{i}^{d}}{d p_{i}}<0$
$q_{i}{ }^{d}=q_{i}^{s}$
$c_{i}=r_{i} w_{i}=y_{i}=p_{i} q_{i}$
$\mathrm{i}=1,2$

Equations (33) to (39) show the equilibrium in the conditions before commencing labor migration and commodity trade, i.e. when neither trade nor migration is yet taking place between the two regions. Now assume that the "real prices" of the two regions are equal and the constraint of trade and migration between two regions is eliminated. We analyze this assumption in the following.

## Changing wage elasticity of price or production elasticity of employment

In the model of equations (33) to (40), assume that the price in region 1 increases for some reason. If both regions begin to exchange labor and commodity, this increase in price will have many effects on the other variables. In addition, changing wage rate will have various effects in the two regions.

According to equations (27), (28), and (39), it is the "real price", which determines the ratio of employment to production in our model. For this reason, we base our analyses on the change of "real price".

Consider a situation, where the wage elasticity of price and production elasticity of employment equal to one; i.e., when changes of $p$ and $r$ as well as $q$ and $w$ are the same. In this case, the real price $\frac{p_{i}}{r_{i}}$ in both regions is constant and the same as when we considered them before. The labor demand (34) and commodity supply (36) in the two regions will not change. Equations (33) and (35) will maintain the equilibrium in the previous position of labor market; and equations (37) and (38) will hold the equilibrium in previous position of commodity market in both regions. Moreover, when real price resulted from changing price and wage rate is constant, the variations in nominal income and nominal consumption of the two regions will not impact on the real rates of income and consumption (39), because $\frac{p_{2}}{r_{2}}=\frac{p_{1}}{r_{1}}$. Added to that, no movement of commodity and labor will occur between the two regions.

Consider another situation, in which $\eta>1$. In other words, the average changes of price to average changes of wage is greater than one. Besides that, assume that both markets in the two regions are in equilibrium. Consider that in region 1 where $\eta>1$, the "real price" increases. Now, we want to observe what changes will appear in the equilibrium state between the two regions.

If we consider region 1 alone, the equilibrium in the labor and commodity market will reach a new point. In fact, the labor demand (34) will be more than the labor supply (33), and equation (35) will establish the equilibrium at a higher wage rate. The degree of this increase depends on the slope and elasticity of curves (33) and (34). In the commodity market, supply (36) increases and demand (38) decreases. The equilibrium in this market (38) causes price to decrease to lower than the amount it was increased to and in fact, the equilibrium will be positioned with a less consumption. The degree of this change is based on the elasticity of the related curves, and also the amount of changes in the nominal income $y_{1}$ because of the increase in $r_{1}$. Final equilibrium occurs when ratio of employment to production equals to "real price".

Now, we consider region 2 in relation to region 1. Assume that $\mathrm{r}_{1}=\mathrm{r}_{2}$ and $\frac{p_{1}}{r_{1}}>\frac{p_{2}}{r_{2}}$; i.e., the "real price" in region 1 is higher than that region 2. Increasing labor demand in region 1 leads to migration of workers from region 2 to 1 . This causes the wage rate of region 1 to be decreased and equilibrium will be reached in labor market of region 1 at higher employment amount. Moreover, increasing the real price in region 1 $\left(\frac{p_{1}}{r_{1}}\right)$ will increase supply and decrease demand for commodity in region 1 . On the other hand, lower prices in region 2 lead to exportation of commodities from region 1 to 2 . Importation of commodities to region 2 will lead to the price reduction in region 2 , and migration of worker leads to rise in wage rate. In such a situation, equilibrium occurs when surpluses of supply and demand for labor and commodity in both regions are equal to each other. In this case, the real price in both regions will be the same. Finally, the general equilibrium in the two regions will be as follows:
$w_{1}^{s}+w_{2}^{s}=w_{1}^{s}(r)+w_{2}^{s}(r)$

$$
\begin{align*}
& w_{1}^{d}+w_{2}^{d}=w_{1}^{d}\left(\frac{p}{r}\right)+w_{2}^{d}\left(\frac{p}{r}\right)  \tag{42}\\
& w_{1}^{d}-w_{1}^{s}=w_{2}^{s}-w_{2}^{d}  \tag{43}\\
& q_{1}^{s}+q_{2}^{s}=q_{1}^{s}\left(\frac{p}{r}\right)+q_{2}^{s}\left(\frac{p}{r}\right)  \tag{44}\\
& q_{1}^{d}+q_{1}^{d}=\frac{y_{1}+y_{2}}{p}  \tag{45}\\
& q_{1}^{s}-q_{1}^{d}=q_{2}^{d}-q_{2}^{s}  \tag{46}\\
& c_{1}+c_{2}=r\left(w_{1}+w_{2}\right)=y_{1}+y_{2}=p\left(q_{1}+q_{2}\right) \tag{47}
\end{align*}
$$

After exchanging labor and commodity, the prices and wage rates of both regions will become equal to each other. In such a situation, the equilibrium condition of equality of the ratio of employment to production and the "real price" can be observed in equation (47). Hence, we will have:
$\frac{\left(w_{1}+w_{2}\right)}{\left(q_{1}+q_{2}\right)}=\frac{p}{r}$
Equation (43) shows the equality of the surplus labor demand in region 1 with the surplus labor supply in region 2 , which is indicative of migration from region 2 to 1 . Equation (46) presents the surplus supply in region 1 and the surplus demand in region 2 . In other words, it displays the equilibrium between the exports from region 1 to 2 and the imports from region 2 to 1 .

When $\eta<1$, the above analyses can be carried out similar to when $\eta>1$, and they are just inverse. Therefore, they have not been restated.

## The policy of no labor and commodity movement

In the above analyses, it was observed that region 2 tends to send migrants and import commodities. If region 2 decides to take a policy, in which no migration to the outside and no importation of commodities into this region occur, what measures should be taken place? The reason behind migration can be attributed to the lower real income, and the reason for commodity import can be attributed to their lower prices. In other words, $\mathrm{y}_{2}<\mathrm{y}_{1}$, or:
$p_{2} q_{2}<p_{1} q_{1} \quad, \quad r_{2} w_{2}<r_{1} w_{1}$
If we consider the values of $u_{2}$ and $v_{2}$ respectively for compensation of the differences of wage and price with region 1 (per unit of labor and commodity), according to inequalities of (49), we will have:
$\left(p_{2}+u_{2}\right) q_{2}=p_{1} q_{1}$ and $\left(r_{2}+v_{2}\right) w_{2}=r_{1} w_{1}$
Which can be written as:
$u_{2}=\frac{p_{1} q_{1}-p_{2} q_{2}}{q_{2}}$ and $v_{2}=\frac{r_{1} w_{1}-r_{2} w_{2}}{w_{2}}$
In other words, if the central government of region 2 pays the subsidy $\mathrm{v}_{2}$ to each labor unit and receives
the tax $u_{1}$ from each production unit, it can set the real income of region 2 equal to that of region 1 and thereby, prevents labor and commodity movements.

To prove this, we should show that in this case, after paying subsidies and receiving taxes, equilibrium will be reached, which is equal to the equilibrium condition in region 1 . In region 2 , the equilibrium condition is as follows:
$\frac{\left(p_{2}+u_{2}\right)}{\left(r_{2}+v_{2}\right)}=\frac{w_{2}}{q_{2}}$
Using equation (50), it can be rewritten as:

$$
\begin{equation*}
\frac{p_{1} q_{1} / q_{2}}{r_{1} w_{1} / w_{2}}=\frac{w_{2}}{q_{2}} \tag{53}
\end{equation*}
$$

In other words:
$\frac{p_{1}}{r_{1}}=\frac{w_{1}}{q_{1}}$
Which is the condition of equality of the ratio of employment to production and the "real price" in region 1.

The financial burden for the government of region 2 for implementing this policy is equal to the tax levy minus the paid subsidy in this case. In other words, if we consider this rate as $\mathrm{G}_{2}-\mathrm{T}_{2}$, we will have:
$G_{2}-T_{2}=u_{2} q_{2}-v_{2} w_{2}=\left(p_{1} q_{1}-p_{2} q_{2}\right)-\left(r_{1} w_{1}-r_{2} w_{2}\right)$
I.e., we can write:
$G_{2}-T_{2}=p_{1} q_{1}-r_{1} w_{1}-\left(p_{2} q_{2}-r_{2} w_{2}\right)$
Since from equation (39) we have:
$r_{i} W_{i}=y_{i}=p_{i} q_{i} \quad, \quad i=1,2$
Therefore, the financial load for the government for implementing this policy will be equal to zero.
$G_{2}-T_{2}=0$
It should be stated that this is the policy of the government of region 2, prior to exchanging the labor and commodity. At this time, the prices and wages in the two regions are still different, and the equilibrium in the markets of region 1 is resulted from equations (33) to (39) for $i=1$. For region 2 , it will be as follows:

$$
\begin{align*}
& w_{2}^{s}=w_{2}^{s}\left(r_{2}+v_{2}\right)  \tag{58}\\
& w_{2}^{d}=w_{2}^{d}\left(\frac{p_{2}+u_{2}}{r_{2}+v_{2}}\right) \tag{59}
\end{align*}
$$

$$
\begin{align*}
& w_{2}^{d}=w_{2}^{s}  \tag{60}\\
& q_{2}^{s}=q_{2}^{s}\left(\frac{p_{2}+v_{2}}{r_{2}+v_{2}}\right)  \tag{61}\\
& q_{2}^{d}=\frac{y_{2}}{p_{2}+u_{2}}  \tag{62}\\
& q_{2}^{d}=q_{2}^{s}  \tag{63}\\
& c_{2}=\left(r_{2}+v_{2}\right) w_{2}=y_{2}=\left(p_{2}+u_{2}\right) q_{2} \tag{64}
\end{align*}
$$

If the central government of region 1 decides to counteract and revert the conditions into the previous state (i.e., when the government of region 2 had not interfered yet), it can again pay subsidies to workers, legislate taxes on selling commodities, and thereby establish the conditions of equation (49), which initiates the migrations to region 1 and exportations from it. In other words, it considers the values of $u_{1}$ and $\mathrm{v}_{1}$ respectively for the compensation of the differences of wage and price with region 2 per unit of labor and commodity. Thus, we have:
$\left(p_{2}+u_{2}\right) q_{2}<\left(p_{1}+u_{1}\right) q_{1},\left(r_{2}+v_{2}\right) w_{2}<\left(r_{1}+v_{1}\right) w_{1}$
If we select $v_{1}$ and $u_{1}$ in a way that:
$u_{2} q_{2}=u_{1} q_{1} \quad, \quad v_{2} w_{2}=v_{1} w_{1}$
We will once again have the inequality (49), and migration from region 2 to 1 as well as commodity export from region 1 to 2 will start over.

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[^0]:    ${ }^{1}$ Professor of Economics, bijan@bidabad.com http://www.bidabad.com
    ${ }^{2}$ MBA Department, Management Faculty, Multimedia University, Malaysia, mahshidsherafati@yahoo.com

